7.2 Compound Interest

Compound Interest - when interest is earned on top of interest

eg. Invest $1000 at 4% for 5 years
Interest paid at the end of each year and added to the investment.

Year 1 $A = 1000(1+0.04)^1$
$A = \$1040$

Year 2 $A = 1040(1+0.04)$
OR
$1000(1+0.04)^2$

Year 3 $A = 1000(1+0.04)^3$

Year 5 $A = 1000(1+0.04)^5$

$$A = P(1+i)^t$$

A = total amount
P = principal
i = annual interest rate
t = time (yrs)

ex. 1 You borrow $8000 at 7.5% compounded yearly for 5 years. How much will you owe?

$A = P(1+i)^t$
$= 8000(1+0.075)^5$
$A = \$11,485.03$

Investments and loans can earn interest at different frequencies.

Investments and loans ... at different frequencies.

annually → once/year

semi annually → twice/year

quarterly → 4x/year

monthly → 12x/year

(2x/month) semi monthly → 2(12) → 24x/year

(52 weeks) bi weekly → 52÷2 → 26x/year

weekly → 52x/year

daily → 365x/year

Adjusted formula for ANY compounding periods

$$A = P\left(1 + \frac{i}{n}\right)^{tn}$$

n = # of compounding periods/year

ex.2 Invest $5000 at 2.5% compounded quarterly for 5 years

"4"

$i = 2.5\% \rightarrow \frac{0.025}{4} = 0.00625 = \frac{i}{n}$ ✓

$t = 5 \rightarrow 4 \times 5 = 20 = nt$ ✓

$A = 5000(1 + 0.00625)^{20}$

$A = \$5663.54$

ex.3 What is the (interest rate) if $6000 is invested for 4 years comp. monthly and earns $1180.89 interest?

? i P

*

and earns $1180.89 interest?

TOTAL AMOUNT = $A = \underset{P}{6000} + \underset{I}{1180.89} = \$7180.89$

$$A = P\left(1 + \frac{i}{n}\right)^{tn}$$

$$\frac{7180.89}{6000} = \frac{6000}{6000}\left(1 + \frac{i}{12}\right)^{4\times 12}$$

$$1.196815 = \left(1 + \frac{i}{12}\right)^{48}$$

$$\sqrt[48]{1.196815} = 1 + \frac{i}{12}$$

$$1.003750011 = 1 + \frac{i}{12}$$

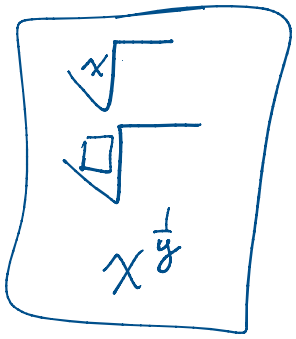
$$-1 \qquad -1$$

$$0.003750011 = \frac{i}{12}$$

$\times 12$

$$0.045000013 = i$$

$$4.5\% = i$$



Rule of 72 – gives an estimate of the time it will take to double your investment

$$\text{Time (years)} = \frac{72}{\text{annual interest rate as a percent}}$$

ex.4 You have $4000 at age 20
You want $16000 at age 32
What interest rate do you need?
Is it doable?

What interest rate ... 0?
Is it doable?
(possible)

$4000 × 2 = 8000 × 2 = $16000
double twice

32 − 20 = 12 years
double in 6 years → $T = \frac{72}{i}$

$6 = \frac{72}{i}$

$i = \frac{72}{6} = 12\%$

Too high for a "small" amount

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