Completing the square ($x^2+bx+c=0$)

FIRST What makes a trinomial a perfect square?

a) $x^2 + 12x + \underline{36}$

$\left(\frac{b}{2}\right)^2 = \left(\frac{12}{2}\right)^2 = 6^2 = 36$

b) $x^2 - 20x + \underline{100}$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-20}{2}\right)^2 = (-10)^2 = 100$

c) $x^2 + 5x + \underline{\frac{25}{4}}$

$\left(\frac{b}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

SECOND How to factor a trinomial square?

a) $x^2 + 6x + 9$

$= (x+3)^2$

$\frac{b}{2}$ OR $\sqrt{c}$

$\frac{6}{2} = \sqrt{9} = 3$

$+$ 6 $\quad$ 3, 3 same

$\times$ 9

$= (x+3)(x+3)$

b) $x^2 + 8x + 16$

$= (x+4)^2$

c) $x^2 - 20x + 100$

$= (x-10)^2$

d) $x^2 - 5x + \frac{25}{4}$

d) $x^2 - 5x + \frac{25}{4}$

$= \left(x - \frac{5}{2}\right)^2$

$\boxed{\text{NOW}}$ complete the square and solve

ex.1 $x^2 + 2x - 3 = 0$

is it a perfect tri. square?

$\left(\frac{2}{2}\right)^2 = 1$ NO

$\boxed{x^2 + 2x + 1} - 1 - 3 = 0$

$+1 - 1 = 0$ no change to equation value

tri. square Factor

$(x+1)^2 - 1 - 3 = 0$

— solve for x —

$(x+1)^2 = 4$

$x+1 = \pm 2$

$x = -1 \pm 2$

$x = -1+2, -1-2$

$x = 1, -3$

$x^2 + 2x + 1 = 3 + 1$

tri. sq. factor

$(x+1)^2 = 4$

$x+1 = \pm 2$

$x = -1 \pm 2$

$x = 1, -3$

ex. 2 $x^2 + 4x - 12 = 0$

$\left(\frac{4}{2}\right)^2 = 4$ complete sq.

$x^2 + 4x + 4 = 12 + 4$

tri. sq. FACTOR

$(x+2)^2 = 16$ solve

$x+2 = \pm 4$

$x = -2 \pm 4$ $\begin{cases} -2+4 = 2 \\ -2-4 = -6 \end{cases}$

$-2-4=-6$

ex. 3 $\quad x^2 - 9x + 7 = 0$

$$x^2 - 9x + \frac{81}{4} = -7 + \frac{81}{4}$$

$$\left(x - \frac{9}{2}\right)^2 = \frac{53}{4}$$

$$x - \frac{9}{2} = \pm\frac{\sqrt{53}}{2}$$

$$x = \frac{9}{2} \pm \frac{\sqrt{53}}{2}$$

$$x = \frac{9 \pm \sqrt{53}}{2}$$

EXACT

$$\left(-\frac{9}{2}\right)^2 = \frac{81}{4}$$

$$\frac{-28}{4} \cancel{\frac{7}{7}} + \frac{81}{4} = \frac{53}{4}$$

$$\boxed{\text{p 223 \# 5, 8, 9, 11a, 12}}$$