

Common log $\rightarrow y = \log_{10} x \rightarrow y = \log x$

often used to solve exponential questions

ex.1 Bacteria increases by splitting in two.

Given 50 bacteria and each splits in two every 30 min, all conditions remain the same, growth rate same.

$$N = 50(2)^{2t}$$

$N = \#$ of bacteria
 $t = \text{time, hrs}$

How long does it take for 51200 bacteria to grow?
 N

$$\frac{51200}{50} = \frac{50}{50} (2)^{2t}$$

$$1024 = 2^{2t}$$

common log.

$$\log 1024$$

$$= \log 2^{2t}$$

power law

$$\log 1024$$

$$= \frac{2t \log 2}{2 \log 2}$$

solve for t

$$(2 \log 2)$$

$$5 \text{ hrs.} = t$$

calc.

A log with base "e" is called the natural log and is written

$$\ln x$$

"lawn x"

$$x > 0$$

$$y = \log_e x, \quad y = \ln x, \quad e^y = x$$

all are equivalent

"e" is a mathematical constant (like π)

approx. 2.718..... (on your calc)

"e" Euler - mathematician

When the number (n) of compounding periods gets infinitely large or there is lots of continuous growth, we use

$$A = A_0 e^{kt}$$

$$\Downarrow$$

$$kt = \log_e \left(\frac{A}{A_0} \right)$$

A_0 - initial amount

t - time period

k - growth factor

$k > 0$ increase/growth

$k < 0$ decrease/decay

ex.2 Solve exponential equation

$$\frac{25}{5} = \frac{5e^{3x}}{5}$$

$$5 = e^{3x}$$

$$3x = \ln 5$$

$$x = \frac{\ln 5}{3}$$

$$x = 0.536$$

$$x = e^y$$

$$\Downarrow$$

$$y = \ln x$$

3. Solve log equation

$$2 \ln(x) + \ln 9 = 18$$

$$x > 0$$

power law
product law

$$\ln x^2 \oplus \ln 9 = 18$$

$$\ln 9x^2 = 18$$

$$\frac{9x^2}{9} = \frac{e^{18}}{9}$$

$$x^2 = \frac{e^{18}}{9}$$

$$x = \pm \sqrt{\frac{e^{18}}{9}}$$

$$x = 2701.028$$

$$4. \quad -\ln x + \ln(x-2) = \ln(x+2) - \ln 2x$$

$x > 2$

$x > 0$
 $x-2 > 0 \Rightarrow x > 2$
 $x+2 > 0 \Rightarrow x > -2$
 $2x > 0 \Rightarrow x > 0$

$$\ln(x-2) - \ln x = \ln(x+2) - \ln 2x$$

quotient law
→

$$\ln\left(\frac{x-2}{x}\right) = \ln\left(\frac{x+2}{2x}\right)$$

same
ln

$$2x \left(\frac{x-2}{x}\right) = \left(\frac{x+2}{2x}\right) 2x$$

$$2x - 4 = x + 2$$

$-x + 4$ $-x + 4$

$$x = 6$$

check restriction

5. growth + decay

Population is 32.6 million in 2019.

About 10% of people left over the last 20 yrs.
 If this continues, what is the population in 2034?

① growth factor k ??

$$kt = \ln\left(\frac{A}{A_0}\right)$$

1 2019 - 2019

$$t = 20$$

$$\frac{A}{A_0} = 100 - 10 = 90\% = 0.9$$

still
there

$$k(20) = \ln(0.9)$$

$$k = \frac{\ln(0.9)}{20}$$

$$(2) A = A_0 e^{kt}$$

$$A_0 = 32.6 \text{ million}$$

$$t = 2034 - 2019 = 15$$

$$A = 32.6 e^{15 \left(\frac{\ln 0.9}{20} \right)}$$

$$A = 30.12 \text{ million}$$

in 2034

→ p 488 # 4-8, choice 9 or 10 ←