

7.3 Solving Exponential Equations

Write 8 as a power of 2 $\rightarrow 2^3$

Write 27 " " " " 3 $\rightarrow 3^3$

Write 16^2 " " " " 2 $\rightarrow (2^4)^2 = 2^8$

An exponential equation has a power with an exponent variable eg. $2^x = 8$

When powers with the same base are equal then their exponents are equal.

ex.1 $2^x = 8 \rightarrow$ change 8 to base 2

$2^x = 2^3 \rightarrow$ since the bases are the same
THEN the exponents are equal

$x = 3$

ex.2. $4^x = 256$

$4^x = 4^4$
 $x = 4$

3. $4^x = \frac{1}{8}$

4 & 8 must be same base
"2"

$(2^2)^x = \frac{1}{2^3}$

$2^{2x} = 2^{-3}$

same base

$2x = -3$

$x = -\frac{3}{2}$

4. $27^x = 9^{2x-1}$
 $(3^3)^x = (3^2)^{(2x-1)}$

1.

$$21 = 1$$

$$(3^3)^x = (3^2)^{(2x-1)}$$

$$3^{3x} = 3^{4x-2}$$

$$3x = 4x - 2$$

+2 -3x -3x +2

$$\boxed{2 = x}$$

bases same
then
exponents equal

REMEMBER

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{8} = 8^{\frac{1}{3}}$$

ex. 5

$$2^x = \sqrt[3]{2}$$

$$2^x = 2^{\frac{1}{3}}$$

$$\boxed{x = \frac{1}{3}}$$

bases =
exp =

6.

$$2^x = \underline{\underline{8}} \sqrt{2}$$

$$2^x = 2^3 2^{\frac{1}{2}}$$

$$2^x = 2^{3\frac{1}{2}}$$

$$x = 3\frac{1}{2} \text{ or } \frac{7}{2}$$

exp law
 $3 + \frac{1}{2} = 3\frac{1}{2} \text{ or } \frac{7}{2}$

7.

$$(\sqrt{125})^{2x+1} = \sqrt[3]{625}$$

$$(\sqrt{5^3})^{2x+1} = \sqrt[3]{5^4}$$

$$\left((5^{\frac{3}{2}})^{\frac{1}{2}} \right)^{2x+1} = (5^{\frac{4}{3}})^{\frac{1}{3}}$$

$$(5^{\frac{3}{2}})^{\frac{1}{2}(2x+1)}$$

$$3x + \frac{3}{2}$$

$$\leq \frac{4}{3}$$

base =

$$\begin{array}{lcl}
 \begin{array}{c} \text{base} = \\ \text{exp} = \end{array} & & \\
 5^{3x + \frac{3}{2}} = 5^{\frac{4}{3}} & & \\
 \downarrow & & \downarrow \\
 3x + \frac{3}{2} = \frac{4}{3} & & \\
 -\frac{3}{2} & & -\frac{3}{2} \\
 3x = -\frac{1}{6} \div 3 & & \\
 x = -\frac{1}{6} \times \frac{1}{3} = \boxed{-\frac{1}{18}} & &
 \end{array}$$

$-\frac{84}{63} - \frac{8}{2} \frac{9}{6} = -\frac{1}{6}$

8.

$$\begin{aligned}
 \left(\frac{1}{9}\right)^x &= 3\sqrt{27} \\
 \left(\frac{1}{3^2}\right)^x &= 3^1 \sqrt{3^3} \\
 \left(3^{-2}\right)^x &= 3 \left(3^3\right)^{\frac{1}{2}} \\
 3^{-2x} &= 3^1 \left(3^{\frac{3}{2}}\right) \\
 3^{-2x} &= 3^{\frac{5}{2}} \\
 \downarrow & \quad \downarrow
 \end{aligned}$$

$$\frac{2}{2} + \frac{3}{2} = \frac{5}{2}$$

$$\frac{-2x}{-2} = \frac{5}{2} \div -2$$

$$\boxed{x = -\frac{5}{4}}$$

REALITY QUESTIONS

Compound interest

n, nt

$A = \text{final amount}$

Compound interest

$$A = A_0 \left(1 + \frac{i}{n}\right)^{nt}$$

A = final amount \$
 A_0 = money invested \$
 i = interest rate
 n = compounding periods
 t = time in years.

Exponential - Growth \rightarrow form $\rightarrow y = ak^{bx}$

- Decay

$$\underline{a, b \in \mathbb{R}}$$

growth
 $k^b > 1$
 $k > 0$
decay
 $0 < k^b < 1$

ex. Principal of $\$1500$ invested at 4% quarterly.
When will it be $\$2500$?

$$i = 0.04$$

$$4 = n$$

$$A = A_0 \left(1 + \frac{i}{n}\right)^{nt}$$

$$2500 = 1500 \left(1 + \frac{0.04}{4}\right)^{4t}$$

use the g.c. enter $y_1 = 1500 \left(1 + \frac{0.04}{4}\right)^{4x} - 2500$
and find the X-INTERCEPT

approx $\rightarrow t = \underline{12.83 \text{ yrs}}$

p 415 # 3-6, 9, 10 (choose 4), 12
↑
Reality