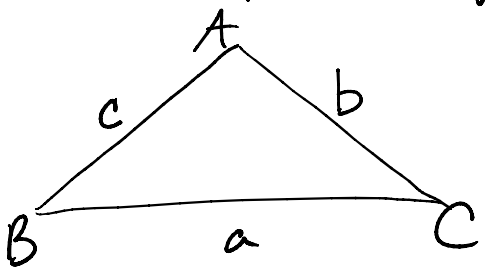
Solving problems in non-right triangles.

In any triangle, the lengths of the sides and the SINES of the angles opposite the sides are <u>proportional</u>.



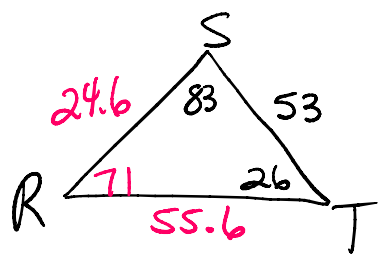
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

OR

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

ex.1 In $\triangle RST$, $\angle S = 83^\circ$, $\angle T = 26^\circ$, $r = 53$

Find the missing sides.



① $\angle R = 180 - 83 - 26 = 71^\circ$

② $$\frac{53}{\sin 71} = \frac{s}{\sin 83} = \frac{t}{\sin 26}$$

$$\frac{53}{\sin 71} = \frac{s}{(\sin 83)}$$

$$\frac{53 \times \sin 83}{\sin 71} = s$$

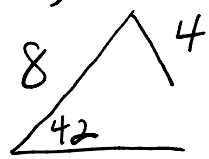
$$\boxed{55.6 = s}$$

$$\frac{53}{\sin 71} = \frac{t}{(\sin 26)}$$

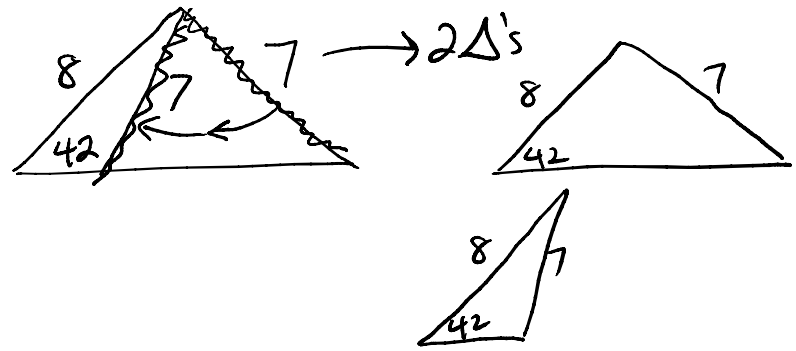
$$\frac{53 \times \sin 26}{\sin 71} = t$$

$$\boxed{24.6 = t}$$

<u>Ambiguous Case</u> — from the given info, 2 sides and an angle opposite one of these sides, the triangle may not exist or there maybe 1 or 2 triangles.



NO Δ



<u>TESTS</u>

① If $\angle A$ acute $(0-90)$ and $a < b$

② $a \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} b \sin A$

$< \rightarrow$ no Δ
$= \rightarrow 1$ Δ
$> \rightarrow 2$ Δ ✓

ex.2 $\triangle ABC$, $\angle A = 30$, $a = 24$, $b = 42$

a) how many solutions? How many triangles?

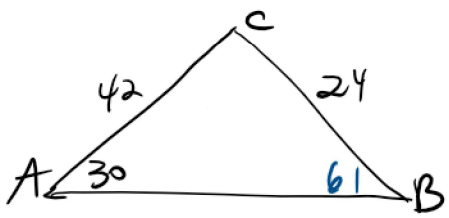
acute ✓
$a < b$ ✓ $\longrightarrow$ $a \begin{matrix} < \\ = \\ > \end{matrix} b \sin A$

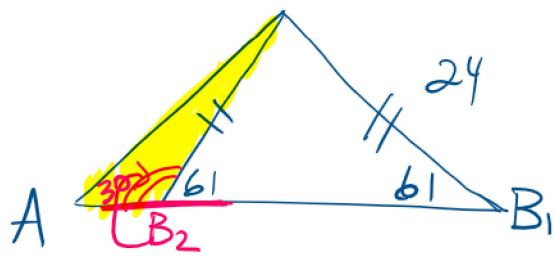
$a \downarrow 24$

$b \sin A \downarrow 42 \sin 30$
21

2 Δ's exist

b) Find <u>all angles</u> to nearest degree





$$\frac{\sin 30}{24} = \frac{\sin B}{(42)}$$

$$\frac{\sin 30 \times 42}{24} = \sin B$$

$$0.875\ldots\ldots = \sin B$$

$$\boxed{61 = B_1}$$

B_2 = supplementary = $180 - 61 = 119 = B_2$

$C_2 = 180 - 30 - 119 = 31 = C_2$

$$180 - 30 - 61 = \boxed{89 = C_1}$$

$$\boxed{\text{p}476 \ \#\ 4\text{-}7, 9, 11}$$