Name:

Physics 12
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Graphical Analysis

Block:

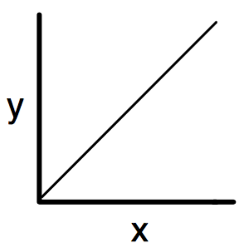
Graphical Analysis is used in labs to
1. establish a relationship between two variables
2. calculate the constant that transforms the relationship into an equation

Establishing a relationship between two variables

The relationship between the variables is shown by the shape of the graph.

Note:
$\propto$ means "proportional to"
k is the constant of proportionality

Linear

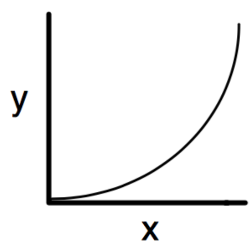


y is directly proportional to *x*

$y \propto x$

Equation: $y = kx$

Power Law (quadratic, cubic, etc.)

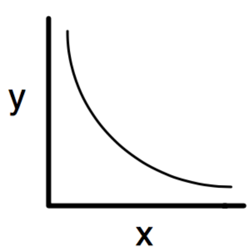


y is directly proportional to the square of *x*

$y \propto x^2$

Equation: $y = kx^2$

Inverse Relationship

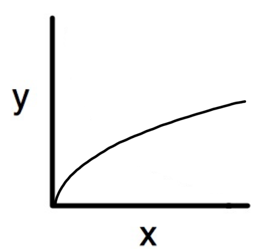


y is inversely proportional to *x*
(or *y* is directly proportional to 1/*x*)

$y \propto \frac{1}{x}$

Equation: $y = \frac{k}{x}$

Square Root



y is directly proportional to the square root of *x*

$y \propto \sqrt{x}$

Equation: $y = k\sqrt{x}$

To confirm the relationship, plot *y* vs. f(*x*). If the relationship is correct, the graph of *y* vs. f(*x*) will yield a straight line passing through the origin (or close to it). For example, if you suspect that *y* is proportional to x^2 (*y* vs. *x* is a parabola), plot *y* vs. x^2.

Determine the constant of proportionality

Upon attaining a straight line graph, the constant of proportionality *k* can be calculated by finding the **slope** of the best fit line.

For manual slope calculations:
- Data points should not be used as data points are not necessarily on the best fit line.
- Use a large triangle to calculate the slope for more precision.

Dimensional analysis can be used to determine the units of the slope.

SI Base Units

Base Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of a substance	mole	mol
Luminous intensity	candela	cd

Some SI Derived Units

Derived Quantity	Name	Symbol	Expressed in base units	Expressed in other SI units
Density	kilogram per cubic meter	kg/m^3	kg/m^3	
Electric charge	coulomb	C	A·s	
Electric field	newton per coulomb	N/C	$kg{\cdot}m/A{\cdot}s^3$	V/m
Electric potential difference	volt	V	$kg{\cdot}m^2/A{\cdot}s^3$	W/A, J/C
Electric resistance	ohm	Ω	$kg{\cdot}m^2/A^2{\cdot}s^3$	V/A
Energy, work	joule	J	$kg{\cdot}m^2/s^2$	N·m
Force	newton	N	$kg{\cdot}m/s^2$	
Frequency	hertz	Hz	s^{-1}	
Magnetic field	tesla	T	$kg/A{\cdot}s^2$	Wb/m^2
Magnetic flux	weber	Wb	$kg{\cdot}m^2/A{\cdot}s^2$	V·s
Power	watt	W	$kg{\cdot}m^2/s^3$	J/s
Pressure	pascal	Pa	$kg/m{\cdot}s^2$	N/m^2
Torque	newton meter	N·m	$kg{\cdot}m^2/s^2$	

Determine the physical meaning of the slope

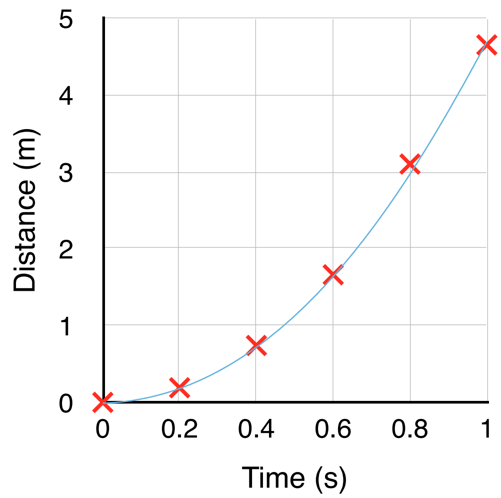
Consider what physical laws would apply and use these laws and equations to determine a theoretical equation which relates the two variables.

Compare the experimental equation to the theoretical equation to provide an interpretation of the slope (*k* = one or more constants).

Once you have provided an interpretation of the slope, you can do the following:
1. If the slope depends on known parameters, you can determine the theoretical value of the slope by using the theoretical value of those parameters.
2. If the slope depends on some physical constant, you can determine the experimental value of that physical constant by using your experimental value of the slope.

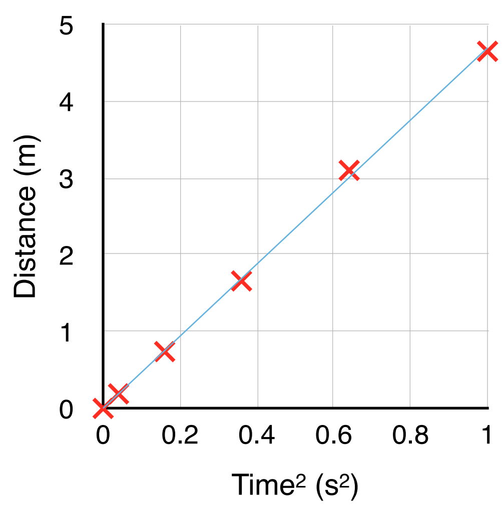
Example
A ball is dropped and the total distance travelled at different times in its fall is measured.

Time (s)	Distance (m)
0	0
0.2	0.19
0.4	0.74
0.6	1.66
0.8	3.10
1.0	4.65



The relationship between distance and time appears parabolic. Verify this by plotting displacement vs. time^2.

Time^2 (s^2)	Distance (m)
0	0
0.04	0.19
0.16	0.74
0.36	1.66
0.64	3.10
1.00	4.65



The plot of displacement vs. time^2 produces a linear graph confirming the relationship.

$$d \propto t^2$$

Calculate the slope to determine the constant of proportionality.

$$d = kt^2$$

$$k = \frac{\Delta d}{\Delta t^2} = \frac{(4.7 - 0.9)\,\mathrm{m}}{(1.0 - 0.2)\,\mathrm{s}^2} = 4.75\frac{\mathrm{m}}{\mathrm{s}^2}$$

Compare the experimental equation to the theoretical equation to interpret the slope *k*.

Experimental equation:

$$d = kt^2$$

Theoretical equation:

$d = v_i t + \frac{1}{2}at^2$ with $v_i = 0$ and $a = g$

$$d = \frac{1}{2}gt^2$$

$$k = \frac{1}{2}g$$

The theoretical value of the slope *k* can be determined by using the theoretical value of *g*.

$$k_{\mathrm{theo}} = \frac{1}{2}g_{\mathrm{theo}} = \frac{1}{2}\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 4.9\frac{\mathrm{m}}{\mathrm{s}^2}$$

The experimental value of *g* can be determined by using the experimental value of *k*.

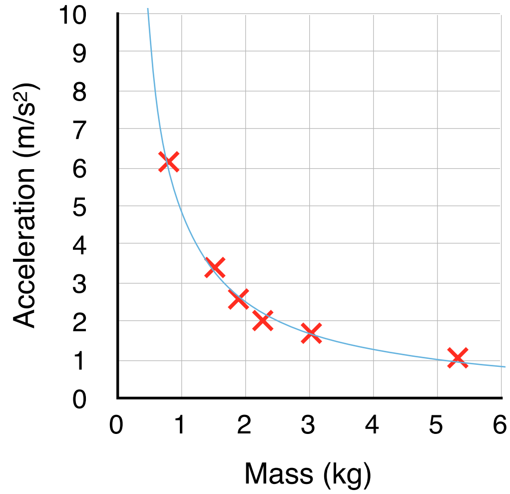
$$k_{\mathrm{exp}} = \frac{1}{2}g_{\mathrm{exp}}$$

$$g_{\mathrm{exp}} = 2k_{\mathrm{exp}} = 2\left(4.75\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 9.5\frac{\mathrm{m}}{\mathrm{s}^2}$$

Example

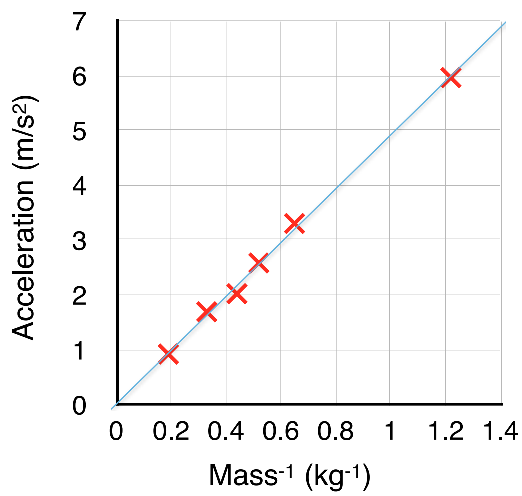
Boxes of varying masses are pushed across a frictionless surface with a constant horizontal force of 5.0 N. The acceleration of each box is measured.

Mass (kg)	Acceleration (m/s^2)
0.82	6.15
1.54	3.40
1.91	2.58
2.29	2.02
3.05	1.69
5.34	1.05



The relationship between acceleration and mass appears to be a reciprocal function. Verify this by plotting acceleration vs. mass^{-1}.

Mass^{-1} (kg^{-1})	Acceleration (m/s^2)
1.22	6.15
0.65	3.40
0.52	2.58
0.44	2.02
0.33	1.69
0.19	1.05



The plot of acceleration vs. mass^{-1} produces a linear graph confirming the relationship.

$$a \propto \frac{1}{m}$$

Calculate the slope to determine the constant of proportionality.

$$a = k\frac{1}{m}$$

$$k = \frac{\Delta a}{\Delta \frac{1}{m}} = \frac{\left(6.9 - 1.0\right)\frac{\text{m}}{\text{s}^2}}{(1.4 - 0.2)\,\frac{1}{\text{kg}}} = 4.92\ \text{kg}\frac{\text{m}}{\text{s}^2} = 4.92\ \text{N}$$

Compare the experimental equation to the theoretical equation to interpret the slope *k*.

Experimental equation:

$$a = k\frac{1}{m}$$

Theoretical equation:

$$a = \frac{F_{\text{net}}}{m}$$

$$k = F_{\text{net}}$$

The theoretical value of the slope can be determined by using the theoretical value of F_{net}.

$$k_{\text{theo}} = F_{\text{theo}} = 5.0\ \text{N}$$

The experimental value of F_{net} can be determined by using the experimental value of *k*.

$$k_{\text{exp}} = F_{\text{exp}}$$
$$F_{\text{exp}} = k_{\text{exp}} = 4.92\ \text{N}$$