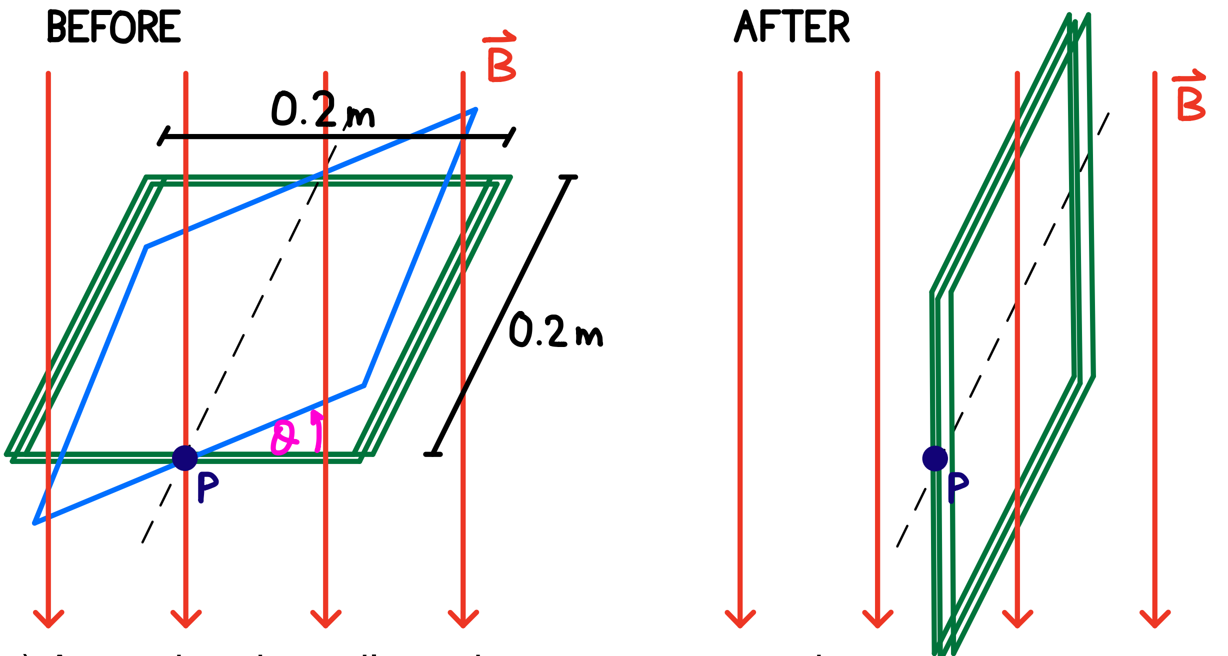
A coil of 25 windings is in a 0.60 T magnetic field. The plane of the coil is initially perpendicular to ***B*** and rotates 90° in 0.04 s so that its plane is parallel to ***B***.

a) What average emf is induced during this time?

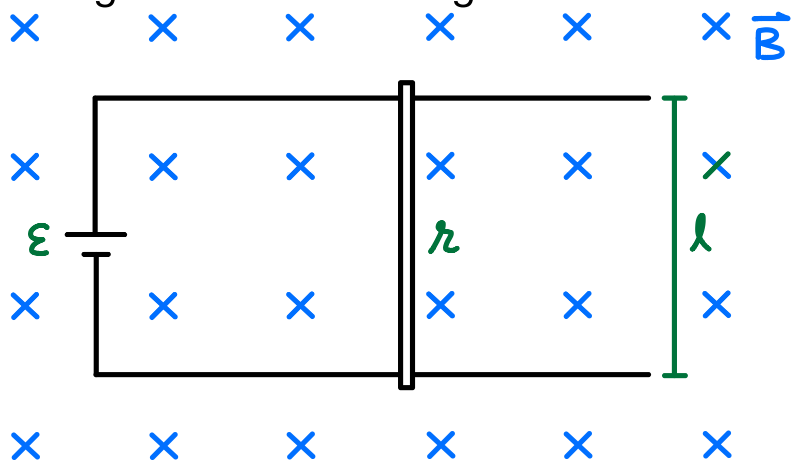
b) In what direction does the current flow at point *P*?



c) Assuming the coil continues to rotate at the same constant rate, derive an equation giving the induced emf as a function of time.

- Determine an expression for the angle the coil makes with the magnetic field as a function of time
- Determine an expression for the magnetic flux as a function of time.
- Apply Faraday's Law (calculus form: $\varepsilon = N\, d\Phi/dt$) to derive an equation for the induced emf as a function of time.

A metal rod of mass m and resistance r is laid across two level conducting rails separated by a distance l. The rod is initially at rest when an applied voltage ε is connected. The entire apparatus is in a uniform magnetic field of strength B.



a) Derive an expression for the current I as a function of the velocity of the rod v.

b) Use Newton's second law to write a differential equation.

c) Solve the differential equation to determine the velocity v, as a function of time t.

d) What is the maximum speed of the rod?

e) Sketch a graph of v vs. t.

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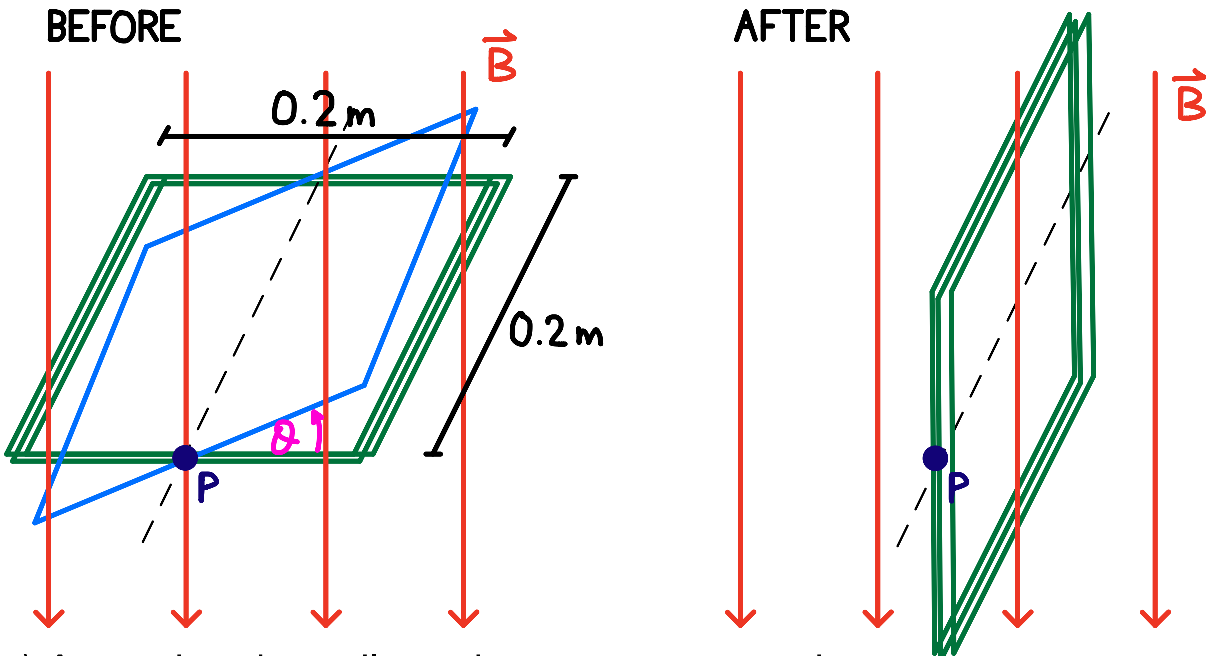

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a) $\mathcal{E} = N\dfrac{\Delta\Phi}{\Delta t}$

$= N\dfrac{\Delta(BA\cos\theta)}{\Delta t}$

$= \dfrac{NBA(\cos\theta_f - \cos\theta_i)}{\Delta t}$ $\quad \theta_f = 90°$, $\cos\theta_i = 1$

$= \dfrac{-NBA}{\Delta t}$

$= \dfrac{-(25)(0.60)(0.2)^2}{0.04}$

$= -\boxed{15\text{ V}}$

b) $\boxed{\text{LEFT}}$

c) i. $\theta(t) = \dfrac{2\pi}{T}t$ $\qquad T = 4\cdot 0.04 = 0.16\text{ s}$

ii. $\Phi(t) = BA\cos(\theta(t))$

$= BA\cos\left(\dfrac{2\pi}{T}t\right)$

iii. $\mathcal{E}(t) = N\dfrac{d\Phi}{dt}$

$= N\dfrac{d}{dt}\left(BA\cos\left(\dfrac{2\pi}{T}t\right)\right)$

$= NBA\dfrac{d}{dt}\left(\cos\left(\dfrac{2\pi}{T}t\right)\right)$

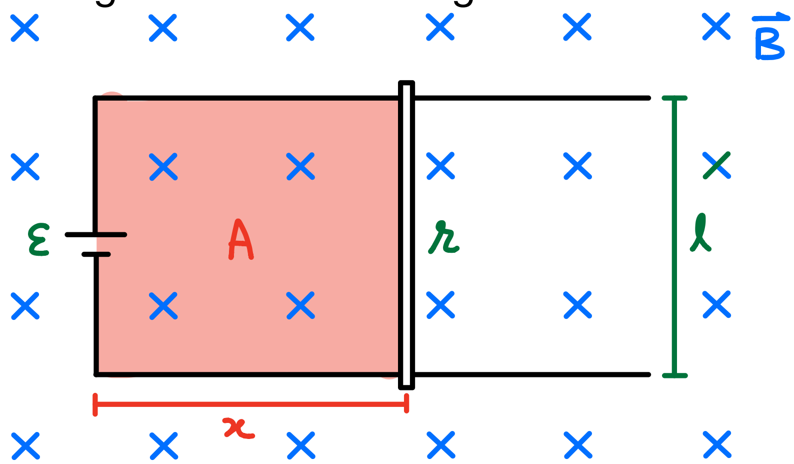
$$= NBA\left(\frac{2\pi}{T}\right)\left(-\sin\left(\frac{2\pi}{T}t\right)\right)$$

$$= -\frac{2\pi NBA}{T}\sin\left(\frac{2\pi}{T}t\right)$$

$$= -\frac{2\pi(25)(0.60)(0.2)^2}{0.16}\sin\left(\frac{2\pi}{0.16}t\right)$$

$$\boxed{\begin{aligned} \mathcal{E}(t) &= -7.5\pi\sin\left(\frac{2\pi}{0.16}t\right) \\ \mathcal{E}(t) &\doteq -23.6\sin\left(\frac{2\pi}{0.16}t\right) \end{aligned}}$$

A metal rod of mass *m* and resistance *r* is laid across two level conducting rails separated by a distance *l*. The rod is initially at rest when an applied voltage ε is connected. The entire apparatus is in a uniform magnetic field of strength *B*.



a) Derive an expression for the current *I* as a function of the velocity of the rod *v*.
b) Use Newton's second law to write a differential equation.
c) Solve the differential equation to determine the velocity *v*, as a function of time *t*.
d) What is the maximum speed of the rod?
e) Sketch a graph of *v* vs. *t.*

a) $\varepsilon_{COUNTER} = N\dfrac{d\Phi}{dt}$

$= B\dfrac{dA}{dt}$

$= B\ell\dfrac{dx}{dt}$

$= B\ell v$

$\varepsilon - \varepsilon_{COUNTER} = Ir$

$I = \dfrac{\varepsilon - \varepsilon_{COUNTER}}{r}$

$$\boxed{I(v) = \frac{\varepsilon - B\ell v}{r}}$$

b) $F_{NET} = ma$

$F_B = ma$

$\ell I B = ma$

$$\boxed{\ell\left(\frac{\varepsilon - \ell B v}{r}\right)B = m\frac{dv}{dt}}$$

c) $\overbrace{\frac{\ell B \varepsilon}{\mathcal{R} m}}^{\alpha} - \overbrace{\frac{\ell^2 B^2}{\mathcal{R} m}}^{\beta} v = \frac{dv}{dt}$

$\alpha = \frac{\ell B \varepsilon}{\mathcal{R} m}$ $\quad \beta = -\frac{\ell^2 B^2}{\mathcal{R} m}$

$$\alpha + \beta v = \frac{dv}{dt}$$

$$dt = \frac{dv}{\alpha + \beta v}$$

$$\int dt = \int \frac{dv}{\alpha + \beta v}$$

$u = \alpha + \beta v$

$du = \beta\, dv \longrightarrow dv = \frac{1}{\beta} du$

$$\int dt = \frac{1}{\beta} \int \frac{du}{u}$$

$$t + c_1 = \frac{1}{\beta} \ln u$$

$$\beta t + c_2 = \ln u$$

$$e^{\beta t + c_2} = u$$

$$c_3 e^{\beta t} = u$$

$$c_3 e^{\beta t} = \alpha + \beta v$$

$$v(t) = c_4 e^{\beta t} - \frac{\alpha}{\beta}$$

$$= c_4 e^{-\frac{\ell^2 B^2}{\mathcal{R} m} t} - \frac{\frac{\ell B \varepsilon}{\mathcal{R} m}}{-\frac{\ell^2 B^2}{\mathcal{R} m}}$$

$$= c_4 e^{-\frac{\ell^2 B^2}{\mathcal{R} m} t} + \frac{\varepsilon}{\ell B}$$

$$v(0) = 0 = c_4 e^{-\frac{\ell^2 B^2}{Rm}(0)} + \frac{\mathcal{E}}{\ell B}$$

$$0 = c_4 + \frac{\mathcal{E}}{\ell B}$$

$$c_4 = -\frac{\mathcal{E}}{\ell B}$$

$$v(t) = -\frac{\mathcal{E}}{\ell B} e^{-\frac{\ell^2 B^2}{Rm} t} + \frac{\mathcal{E}}{\ell B}$$

$$\boxed{v(t) = \frac{\mathcal{E}}{\ell B}\left(1 - e^{-\frac{\ell^2 B^2}{Rm} t}\right)}$$

d) $$v(t) = \frac{\mathcal{E}}{\ell B}\left(1 - \underbrace{e^{-\frac{\ell^2 B^2}{Rm} t}}\right)$$

$e^{-\frac{\ell^2 B^2}{Rm} t} > 0$ FOR ALL t AND HAS A MINIMUM VALUE WHEN $t = \infty$

$$\lim_{t \to \infty} e^{-\frac{\ell^2 B^2}{Rm} t} = 0$$

$$v_{MAX} = \lim_{t \to \infty} v(t) = \lim_{t \to \infty} \frac{\mathcal{E}}{\ell B}\left(1 - e^{-\frac{\ell^2 B^2}{Rm} t}\right)$$

$$= \frac{\mathcal{E}}{\ell B}(1 - 0) = \boxed{\frac{\mathcal{E}}{\ell B}}$$

ALTERNATE SOLUTION:

$$v_{MAX} \longrightarrow a = 0 \longrightarrow F_{NET} = F_B = 0$$

$$\longrightarrow I = 0 \longrightarrow \mathcal{E} = \mathcal{E}_{COUNTER}$$

$$\mathcal{E} = \ell B v_{MAX}$$

$$v_{MAX} = \frac{\mathcal{E}}{\ell B}$$

e)