KINEMATICS IS THE STUDY OF MOTION.

POSITION, DISPLACEMENT AND DISTANCE.

- **POSITION** is a vector describing where an object is relative to an origin.
- **DISPLACEMENT** is the change in position and is represented by a vector joining the initial and final positions.

- **DISTANCE** is the length of a path.
- **DISTANCE** is a scalar.
- If a path taken is straight, the distance is equal to the displacement.
EXAMPLE
A dog walks from corner to corner around a square city block of side length 100 m. Calculate the distance travelled and the displacement.

SPEED AND VELOCITY
- SPEED is the rate at which distance is travelled.
- SPEED is a scalar.

\[ v = \frac{d}{t} \]

\( v \): SPEED (m/s)
\( d \): DISTANCE (m)
\( t \): TIME (s)
**Velocity** is the rate at which an object changes its position. Velocity is a vector.

\[ \vec{v} = \frac{\vec{d}}{t} \]

\[ \vec{v} : \text{velocity} \quad (m/s) \]
\[ \vec{d} : \text{displacement} \quad (m) \]
\[ t : \text{time} \quad (s) \]

If velocity is not constant, this equation gives average velocity.

**Uniform motion** refers to motion at a constant velocity.

**Example**

Light from the Sun takes 8½ minutes to reach Earth. If light travels at \(3.00 \times 10^8 \text{ m/s}\), how far away is the Sun?
**EXAMPLE**

A student completes four laps around a 400 m track in 390 s. What are the student's average speed and velocity?

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**ACCELERATION**

- **ACCELERATION** is the rate of change of velocity.
- **ACCELERATION** is a vector.

\[
\vec{a} = \frac{\Delta \vec{v}}{t} = \frac{\vec{v}_f - \vec{v}_i}{t}
\]

\(\vec{a}\): Acceleration \((\text{m/s}^2)\)

\(\Delta \vec{v}\): Change in velocity \((\text{m/s})\)

\(\vec{v}_f\): Final velocity \((\text{m/s})\)

\(\vec{v}_i\): Initial velocity \((\text{m/s})\)

\(t\): Time \((\text{s})\)
For motion with constant acceleration, the average velocity can be expressed in the following forms:

\[ \bar{v} = \frac{\Delta d}{t} \]

\[ \bar{v} = \frac{\vec{v}_i + \vec{v}_f}{2} \]

\[ \vec{v}_f = \vec{v}_i + \vec{a}t \]

\[ \vec{a} = \left( \frac{\vec{v}_f + \vec{v}_i}{2} \right) t \]

\[ v_f^2 = v_i^2 + 2\vec{a}\vec{d} \]

\[ \vec{d} = \vec{v}_i t + \frac{1}{2} \vec{a}t^2 \]

Vector \( \vec{V} \): Average velocity \( (\text{m/s}) \)

Vector \( \vec{V}_i \): Initial velocity \( (\text{m/s}) \)

Vector \( \vec{V}_f \): Final velocity \( (\text{m/s}) \)

\( \vec{a} \): Acceleration \( (\text{m/s}^2) \)

\( \vec{V}_f \): Final velocity \( (\text{m/s}) \)

\( \vec{V}_i \): Initial velocity \( (\text{m/s}) \)

\( \vec{d} \): Displacement \( (\text{m}) \)

\( t \): Time \( (\text{s}) \)
EXAMPLE
A SPRINTER, STARTING FROM REST, ACCELERATES AT A RATE OF \(1.1 \, \text{m/s}^2\) FOR THE ENTIRETY OF A 100 m DASH. WHAT IS HIS FINAL VELOCITY?

① IDENTIFY YOUR UNKNOWN ALONG WITH THREE GIVENs.

② CHOOSE THE EQUATION WITH THE UNKNOWN AND THREE GIVENs.

③ SIMPLIFY BY PLUGGING IN ANY ZEROS.

④ ALGEBRAICALLY SOLVE FOR THE UNKNOWN.

⑤ PLUG IN VALUES.
EXAMPLE
Rammus is powerballing with an acceleration of $1.4 \text{ m/s}^2$. If he starts from rest and powerballs for 7.0 s, how far does he travel?

EXAMPLE
A police car increases its speed from $50 \text{ km/h}$ to $90 \text{ km/h}$ in 2.5 s. What was its acceleration?
FREE FALL

ACCELERATION DUE TO GRAVITY, \( g \), IS THE ACCELERATION OF FREE FALL DUE TO GRAVITATIONAL FORCE.

ON THE SURFACE OF EARTH, \( g = 9.8 \, \text{m/s}^2 \) DOWN. CAN BE POSITIVE OR NEGATIVE (SEE ① BELOW)

PROBLEM SOLVING TIPS

① BEFORE LISTING YOUR GIVENS, DECIDE ON ONE DIRECTION TO BE POSITIVE. ALL VECTORS IN THE OPPOSITE DIRECTION WILL BE NEGATIVE.

② CAN'T FIND YOUR THREE GIVENS? HERE ARE SOME NOT EXPLICITLY STATED:

- FREE FALL: \( a = 9.8 \, \text{m/s}^2 \) DOWN
- STARTS AT REST: \( v_i = 0 \, \text{m/s} \)
- OBJECT IS Dropped FROM A STATIONARY POSITION: \( v_i = 0 \, \text{m/s} \)
- OBJECT IN FREE FALL REACHES ITS PEAK: \( v_f = 0 \, \text{m/s} \)
OBJECT IN FREE FALL INITIALLY MOVING UPWARDS RETURNS TO ORIGINAL POSITION/HEIGHT: \( v_f = -v_i \).

If you are asked for the velocity at which an object hits the ground, you are looking for the velocity before stopping: \( v_f \neq 0 \)

**Example**

How far will a hammer fall after 2.0 s if it is dropped from rest?
EXAMPLE
A bullet is fired from a gun upwards at 700 m/s. What maximum height will it reach?

EXAMPLE
A boy throws a ball upwards at a speed of 15 m/s. How long does it take to return to his hand?
GRAPHS OF MOTION
POSITION vs. TIME GRAPHS
OR DISPLACEMENT VS. TIME

THE SLOPE OF A \( d \) vs. \( t \) GRAPH IS
EQUAL TO THE VELOCITY.

THE AVERAGE VELOCITY IS THE
SLOPE OF A STRAIGHT LINE
JOINING TWO POINTS ON A \( d \) vs. \( t \)
GRAPH.
EXAMPLE
CALCULATE THE VELOCITY FOR EACH PART OF THE GRAPH.

A CURVE ON A $d$ vs. $t$ GRAPH DESCRIBES ACCELERATION.
FOR A CURVED $d$ vs. $t$ GRAPH, ESTIMATE THE SLOPE USING A TANGENT LINE.
THE SLOPE OF A TANGENT LINE ON A $d$ vs. $t$ GRAPH IS THE INSTANTANEOUS VELOCITY.
EXAMPLE

Determine the instantaneous velocity at 4.0 s and the average velocity from 0 to 8.0 s.
VELOCITY vs. TIME GRAPHS

- Constant Positive Acceleration (speeding up forwards)
- Constant Velocity
- Constant Negative Acceleration (slowing down forwards)

The slope of a v vs. t graph is equal to the acceleration. The area under a v vs. t graph is equal to the displacement.
EXAMPLE

USE THE GRAPH TO CALCULATE THE FOLLOWING:

a) THE ACCELERATION AT $t=4\text{s}$

b) THE AVERAGE ACCELERATION FROM 0 TO 14\text{s}

c) THE TOTAL DISPLACEMENT

d) THE TOTAL DISTANCE TRAVELLED

e) THE AVERAGE VELOCITY FROM 0 TO 14\text{s}

f) THE AVERAGE SPEED FROM 0 TO 14\text{s}
PROJECTILE MOTION

PROJECTILE MOTION is any sort of free fall motion that has a horizontal component of velocity.

**Example**

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\[ \vec{v} \]
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\[ \vec{a} = g \]

The motion of a projectile can be analyzed by looking at the horizontal and vertical components separately. Each direction is independent of the other.

The horizontal component follows uniform motion \((v_x = \text{constant})\).
\[ d_x = v_x t \]

**THE VERTICAL COMPONENT FOLLOWS UNIFORMLY ACCELERATED MOTION**

\( a_y = 9.8 \text{ m/s}^2 \) **DOWN**.

\[ v_{y_f} = v_{y_i} + a_y t \]
\[ d_y = \left( \frac{v_{y_i} + v_{y_f}}{2} \right) t \]
\[ v_{y_f}^2 = v_{y_i}^2 + 2a_y d_y \]
\[ d_y = v_{y_i} t + \frac{1}{2} a_y t^2 \]

**TIME LINKS THE HORIZONTAL AND VERTICAL MOTION.**
EXAMPLE
A soccer ball is kicked off of a 100 m cliff with a velocity of $20 \frac{m}{s}$ to the right.

a) How long will it spend in the air?
b) How far from the base of the cliff will it land?
c) What will be its velocity right before it hits the ground?

\[ v = 20 \frac{m}{s} \]

\[ d_y = 100 \, m \]

\[ \begin{array}{c|c}
\text{HORIZONTAL} & \text{VERTICAL} \\
\hline
v_x = 20 \frac{m}{s} & v_y_i = 0 \\
t = ? & a_y = +9.8 \frac{m}{s^2} \\
d_x = ? & d_y = +100 \, m \\
t = ? & t = ? \end{array} \]
\[ d = v_i t + \frac{1}{2} a t^2 \]

\[ d = \frac{1}{2} a t^2 \]

\[ t^2 = \frac{2d}{a} \]

\[ t = \sqrt{\frac{2d}{a}} \]

\[ = \sqrt{\frac{2(100)}{9.8}} \]

\[ = 4.5175 \text{ s} \rightarrow 4.5 \text{ s} \]

b) \[ v_x = 20 \frac{m}{s} \]
\[ t = 4.5175 \text{ s} \]
\[ d_x = ? \]

\[ d = vt \]
\[ = (20)(4.5175) \]
\[ = 90.3508 \text{ m} \]

\[ \rightarrow 90. \text{ m} \]

c) \[ v_x = 20 \frac{m}{s} \]
\[ t = 4.5175 \text{ s} \]
\[ d_x = 90.3508 \text{ m} \]

\[ v_{yt} = 0 \]
\[ a_y = +9.8 \frac{m}{s^2} \]
\[ d_y = +100 \text{ m} \]
\[ t = 4.5175 \text{ s} \]

\[ v_{yt} = ? \]
\[ v_{yi} = 0 \]
\[ v_f^2 = v_i^2 + 2ad \]
\[ v_f^2 = 2ad \]
\[ v_f = \sqrt{2ad} \]
\[ = \sqrt{2(9.8)(100)} \]
\[ = +44.2719 \text{ m/s} \]
\[ \text{(DOWN)} \]

\[ v_x = 20 \text{ m/s} \text{ RIGHT} \]
\[ v_{xf} = 44.2719 \text{ m/s} \text{ DOWN} \]

\[ v_f^2 = v_x^2 + v_{yf}^2 \]
\[ v_f = \sqrt{v_x^2 + v_{yf}^2} \]
\[ = \sqrt{(20)^2 + (44.2719)^2} \]
\[ = 48.5798 \text{ m/s} \]

\[ \tan \theta = \frac{v_{yf}}{v_x} \]
\[ \theta = \tan^{-1} \left( \frac{v_{yf}}{v_x} \right) \]
\[ = \tan^{-1} \left( \frac{44.2719}{20} \right) \]
\[ = 65.69^\circ \]

49 \text{ m/s} \quad 66^\circ \text{ BELOW THE HORIZONTAL}
EXAMPLE
A BALL IS THROWN FROM LEVEL GROUND WITH A VELOCITY OF 12 m/s
60° ABOVE THE HORIZONTAL.
    a) HOW LONG WILL IT SPEND IN THE AIR?
    b) HOW FAR FROM THE THROWER WILL IT LAND?
    c) WHAT WILL BE ITS VELOCITY RIGHT BEFORE IT HITS THE GROUND?