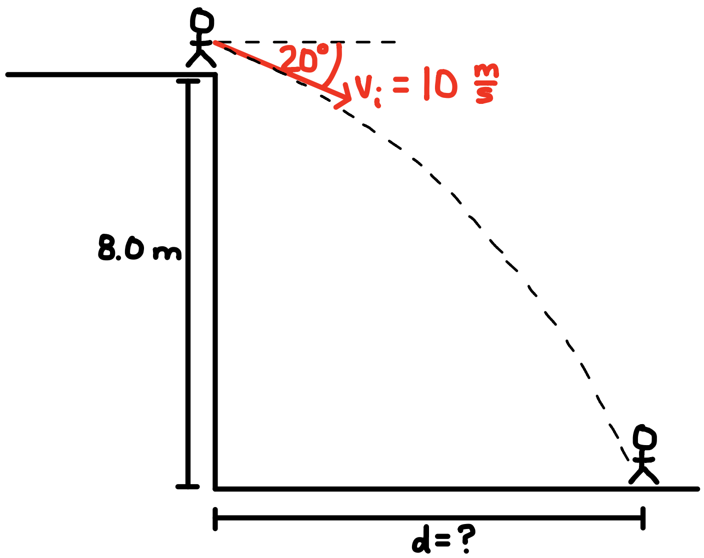
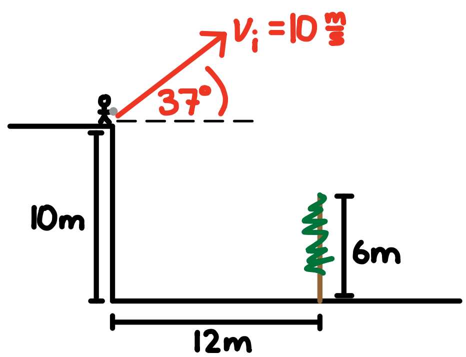
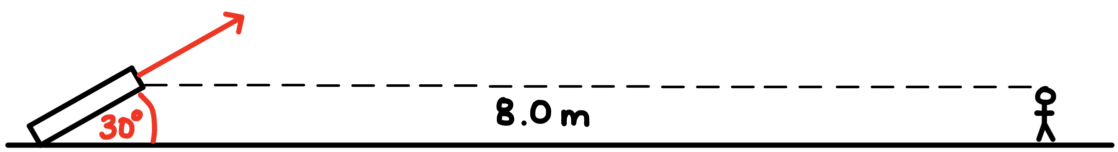
Anna throws a ball from a 8.0 m high tower towards Bobby at a velocity of 10 m/s 20º below the horizontal. How far from the tower should Bobby stand to catch the ball?



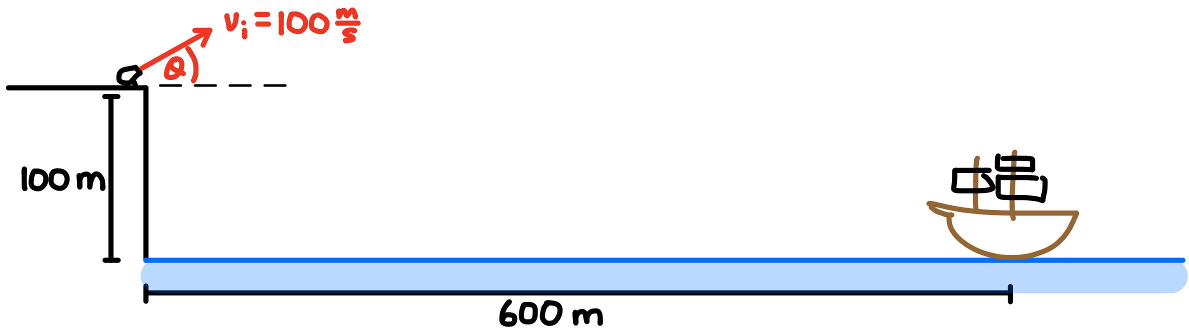
A rock is thrown from the top of a 10.0 m high building at a velocity of 10 m/s 37° above the horizontal. A 6.0 m tall tree is 12.0 m from the building. Will the rock make it over the tree? If it does, at what speed does it hit the ground? If it doesn't at what speed does it hit the tree?



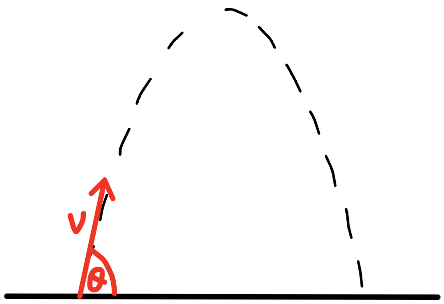
A launcher angled at 30º as shown is used to fire a ball towards a student 8.0 m away. At what speed must the ball be fired in order to hit the student?



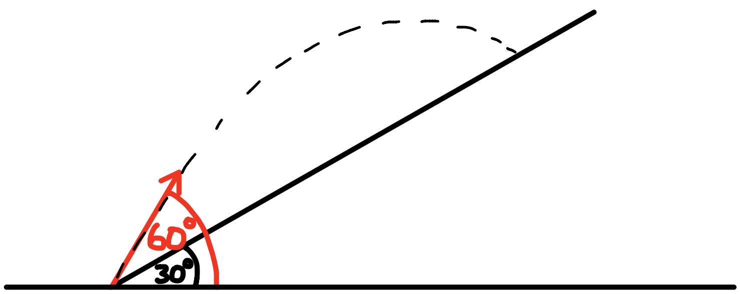
A cannon on a 100 m high cliff is fired at a speed of 100 m/s towards an enemy ship 500 m from the base of the cliff. At what angle must the cannon be launched in order to hit the ship?



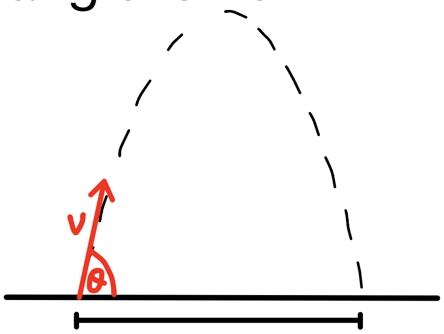
At 1/2 its maximum height (relative to its initial position), the speed of a projectile is 3/4 of its initial speed. What was its initial launch angle?



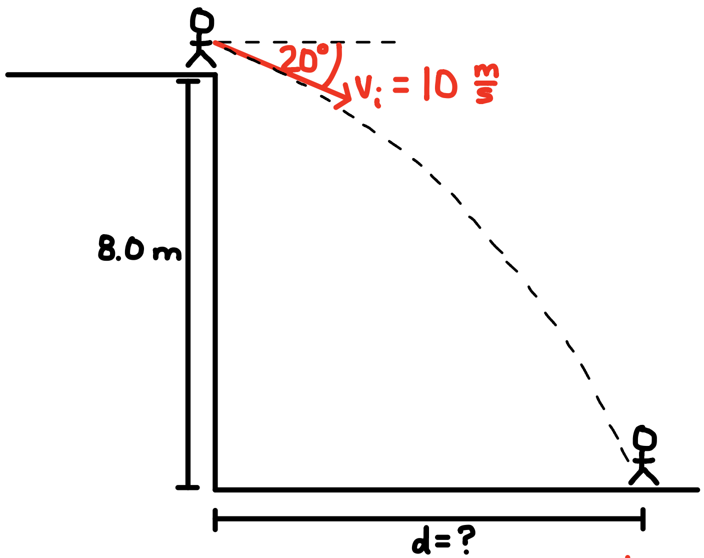
A projectile is fired at the bottom of an incline with a velocity of 100 m/s at an angle of 60º above the horizontal. The incline has an angle of elevation of 30º. What horizontal distance does the projectile travel before impacting the incline?



Show with calculus that the range of a projectile launched on level ground is a maximum when the angle is 45°.



Anna throws a ball from a 8.0 m high tower towards Bobby at a velocity of 10 m/s 20° below the horizontal. How far from the tower should Bobby stand to catch the ball?




| X | Y |
| --- | --- |
| $V_x = 10\cos 20^\circ\ \frac{m}{s}$ | $V_{iy} = +10\sin 20^\circ\ \frac{m}{s}$ |
| $t = ?$ | $a_y = +9.8\ \frac{m}{s^2}$ |
| $d_x = ?$ | $d_y = +8.0\ m$ |
| | $t = ?$ |

$\downarrow +$

$$d_y = v_{iy}t + \frac{1}{2}a_y t^2$$

$$0 = \frac{1}{2}a_y t^2 + v_{iy}t - d_y$$

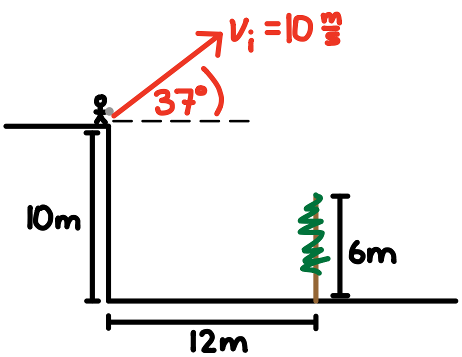
$$t = \frac{-v_{iy} \pm \sqrt{v_{iy}^2 - 4(\frac{1}{2}a_y)(-d_y)}}{2(\frac{1}{2}a_y)}$$

$$t = \cancel{-1.6736\ s},\ 0.9756\ s$$

$$d_x = v_x t$$

$$= 9.17\ m$$

A rock is thrown from the top of a 10.0 m high building at a velocity of 10 m/s 37° above the horizontal. A 6.0 m tall tree is 12.0 m from the building. Will the rock make it over the tree? If it does, at what speed does it hit the ground? If it doesn't at what speed does it hit the tree?




| X | Y |
|---|---|
| $V_x = 10 \cos 37° \frac{m}{s}$ | $V_{iy} = 10 \sin 37° \frac{m}{s}$ |
| $d_x = 12$ m | $a_y = -9.8 \frac{m}{s^2}$ |
| $t = ?$ | $t = ?$ |
| | $d_y = ?$ |
| $t = \frac{d_x}{V_x} = \frac{12}{10 \cos 37°} = 1.503s$ | |
| | $d_y = v_{iy}t + \frac{1}{2}a_y t^2$ |
| | $= (10 \sin 37°)(1.503) + \frac{1}{2}(-9.8)(1.503)^2$ |
| | $= -2.02$ m |

THE ROCK MAKES IT OVER THE TREE. (IT CLEARS IT BY ~2 m.)

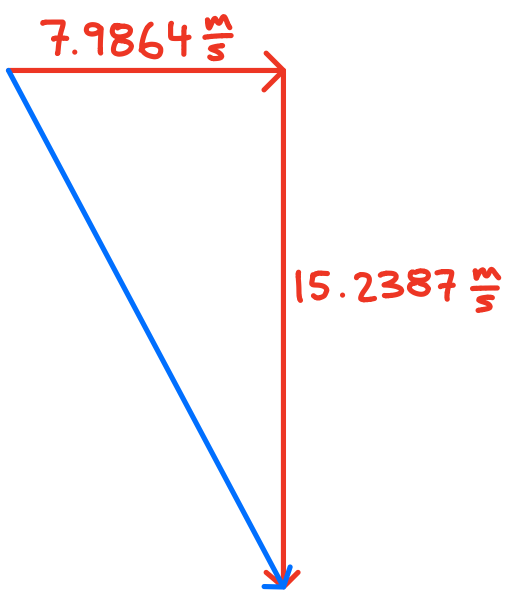
| X | Y |
|---|---|
| $V_x = 10 \cos 37^\circ \ \frac{m}{s}$ | $V_{iy} = 10 \sin 37^\circ \ \frac{m}{s}$ |
| | $a_y = -9.8 \frac{m}{s^2}$ |
| | $d_y = -10 \ m$ |
| | $V_{fy} = \ ?$ |

$$V_{fy}^2 = V_{iy}^2 + 2a_y d_y$$

$$V_{fy} = \pm\sqrt{V_{iy}^2 + 2a_y d_y}$$

$$= \pm\sqrt{(10 \sin 37^\circ)^2 + 2(-9.8)(-10)}$$

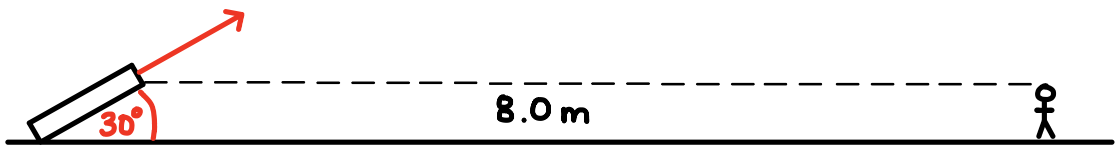
$$= \cancel{\pm}\, 15.2387 \ \frac{m}{s}$$




$$V_f = \sqrt{V_x^2 + V_{fy}^2}$$

$$= \boxed{17.2 \ \frac{m}{s}}$$

A launcher angled at 30° as shown is used to fire a ball towards a student 8.0 m away. At what speed must the ball be fired in order to hit the student?




| X | Y |
| --- | --- |
| $d_x = 8.0\ \text{m}$ | $d_y = 0$ |
| $v_x = v_i \cos 30°$ $(v_i = ?)$ | $a = -9.8\ \frac{m}{s^2}$ |
| $t = ?$ | $v_{iy} = v_i \sin 30°$ $(v_i = ?)$ |
| | $t = ?$ |

+ ↑

$$\boxed{t = \frac{d_x}{v_x} = \frac{d_x}{v_i \cos\theta}}$$

$$d_y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$0 = v_i \sin\theta\, t + \frac{1}{2} a_y t^2$$

$$0 = t\underbrace{\left(v_i \sin\theta + \frac{1}{2} a_y t\right)}$$

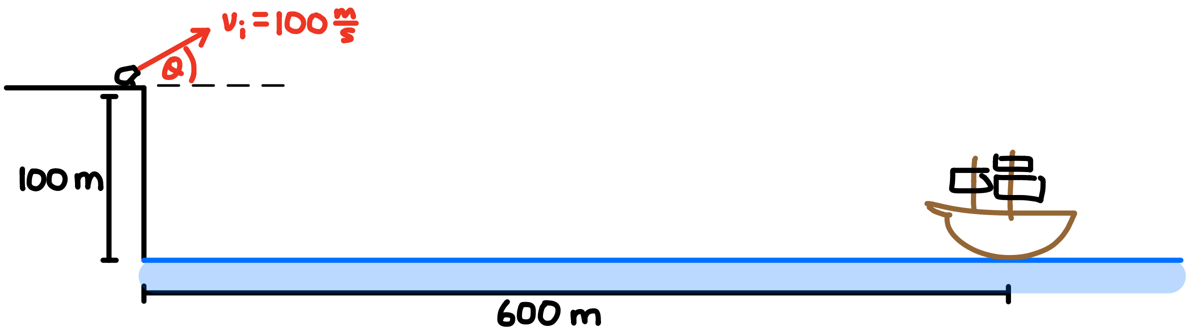
$$v_i \sin\theta + \frac{1}{2} a_y t = 0$$

$$\boxed{t = \frac{-2 v_i \sin\theta}{a_y}}$$

$$\frac{d_x}{v_i \cos\theta} = -\frac{2 v_i \sin\theta}{a_y}$$

$$v_i = \sqrt{-\frac{d_x a_y}{2 \sin\theta \cos\theta}} = 9.51\ \frac{m}{s}$$

A cannon on a 100 m high cliff is fired at a speed of 100 m/s towards an enemy ship 500 m from the base of the cliff. At what angle must the cannon be launched in order to hit the ship?




| x | y |
| --- | --- |
| $V_x = V_i \cos\theta = 100 \cos\theta\ \frac{m}{s}$ | $V_{iy} = V_i \sin\theta = 100 \sin\theta\ \frac{m}{s}$ |
| $d_x = 600\ m$ | $a_y = -9.8\ \frac{m}{s^2}$ |
| $t = ?$ | $d_y = -100\ m$ |
| | $t = ?$ |

$$t = \frac{d_x}{V_x} = \frac{d_x}{V_i \cos\theta}$$

$$d_y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$d_y = v_i \sin\theta \left(\frac{d_x}{v_i \cos\theta}\right) + \frac{1}{2} a_y \left(\frac{d_x}{v_i \cos\theta}\right)^2$$

$$d_y = d_x \tan\theta + \frac{a_y d_x^2}{2 v_i^2} \sec^2\theta \qquad \sec^2\theta = \tan^2\theta + 1$$

$$d_y = d_x \tan\theta + \frac{a_y d_x^2}{2 v_i^2} (\tan^2\theta + 1)$$

$$d_y = d_x \tan\theta + \frac{a_y d_x^2}{2v_i^2}\tan^2\theta + \frac{a_y d_x^2}{2v_i^2}$$

$$0 = \underbrace{\frac{a_y d_x^2}{2v_i^2}}_{a=-176.4}\tan^2\theta + \underbrace{d_x}_{b=600}\tan\theta + \underbrace{\left(\frac{a_y d_x^2}{2v_i^2} - d_y\right)}_{c=-76.4}$$

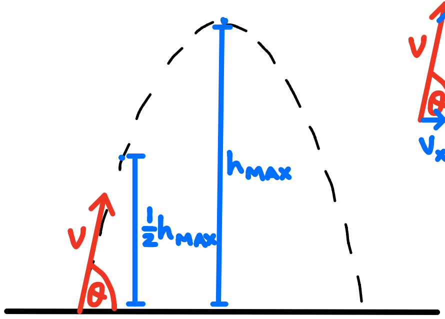
$$\tan\theta = \frac{-600 \pm \sqrt{600^2 - 4(-176.4)(-76.4)}}{2(-176.4)}$$

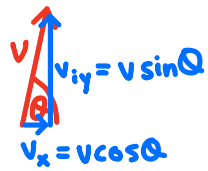
$$\tan\theta = 0.13249,\ 3.26887$$

$$\theta = \tan^{-1}(0.13249) \quad \text{or} \quad \theta = \tan^{-1}(3.26887)$$

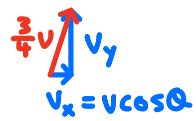
$$\theta = 7.5^\circ \qquad\qquad \theta = 73.0^\circ$$

At 1/2 its maximum height (relative to its initial position), the speed of a projectile is 3/4 of its initial speed. What was its initial launch angle?




AT ½ $h_{MAX}$, $v_x$ IS THE SAME BUT $v_y$ HAS DECREASED.




$$v_y = \sqrt{\left(\tfrac{3}{4}v\right)^2 - (v\cos\theta)^2}$$

$$v_y = v\sqrt{\tfrac{9}{16} - \cos^2\theta}$$

| y (AT ½ $h_{MAX}$) | y (AT $h_{MAX}$) |
|---|---|
| $v_{iy} = v\sin\theta$ | $v_{iy} = v\sin\theta$ |
| $v_{fy} = v\sqrt{\frac{9}{16} - \cos^2\theta}$ | $v_{fy} = 0$ |
| $a = -9.8\ \frac{m}{s^2}$ | $a_y = -9.8\ \frac{m}{s^2}$ |
| $d_y = \frac{1}{2}h_{MAX}$ | $d_y = h_{MAX} = ?$ |

$$\cancel{v_{fy}}^{\,0\,2} = v_{iy}^2 + 2a_y d_y$$

$$d_y = \frac{-v_{iy}^2}{2a_y}$$

$$h_{MAX} = -\frac{v^2\sin^2\theta}{2a_y}$$

$$v_{fy}^2 = v_{iy}^2 + 2a_y d_y$$

$$\cancel{v^2}\left(\frac{9}{16} - \cos^2\theta\right) = \cancel{v^2}\sin^2\theta + 2\cancel{a_y}\left(-\frac{1}{2}\,\frac{\cancel{v^2}\sin^2\theta}{\cancel{2a_y}}\right)$$

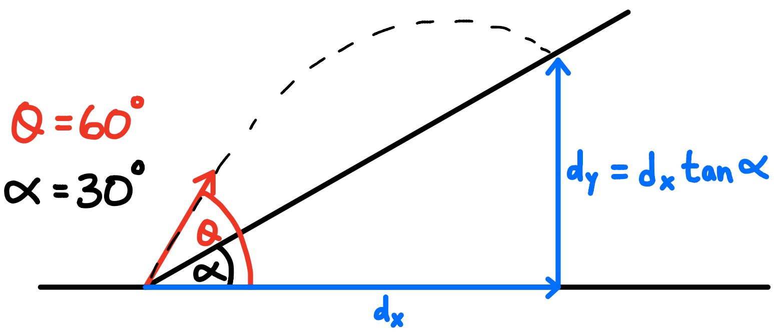
$$\frac{9}{16} - (1 - \sin^2\theta) = \sin^2\theta - \frac{\sin^2\theta}{2}$$

$$-\frac{7}{16} + \sin^2\theta = \sin^2\theta - \frac{\sin^2\theta}{2}$$

$$\sin\theta = \sqrt{\frac{7}{8}}$$

$$\theta = \sin^{-1}\left(\sqrt{\frac{7}{8}}\right) = 69.3°$$

A projectile is fired at the bottom of an incline with a velocity of 100 m/s at an angle of 60º above the horizontal. The incline has an angle of elevation of 30º. What horizontal distance does the projectile travel before impacting the incline?




| x | y |
|---|---|
| $V_x = V_i \cos\theta = 100 \cos 60° \frac{m}{s}$ | $V_{iy} = V_i \sin\theta = 100 \sin 60° \frac{m}{s}$ |
| $d_x = ?$ | $a_y = -9.8 \frac{m}{s^2}$ |
| $t = ?$ | $d_y = d_x \tan\alpha$ |
| | $t = ?$ |

$$t = \frac{d_x}{V_x} = \frac{d_x}{V_i \cos 60°}$$

$$d_y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$d_x \tan\alpha = v_i \sin\theta \left(\frac{d_x}{v_i \cos\theta}\right) + \frac{1}{2} a_y \left(\frac{d_x}{v_i \cos\theta}\right)^2$$

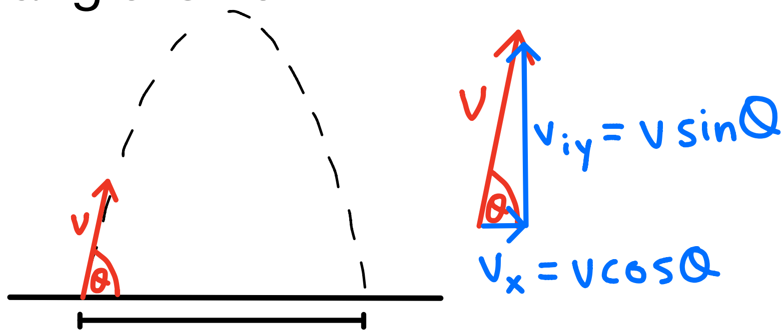
$$0 = \frac{a_y}{2 v_i^2 \cos^2\theta} d_x^2 + (\tan\theta - \tan\alpha) d_x$$

$$0 = d_x \left(\frac{a_y}{2 v_i^2 \cos^2\theta} d_x + \tan\theta - \tan\alpha\right)$$

$$d_x = 0, \ \frac{2 v_i^2 \cos^2\theta}{a_y} (\tan\alpha - \tan\theta)$$

$$d_x = 0, \ 589 \text{ m}$$

Show with calculus that the range of a projectile launched on level ground is a maximum when the angle is 45º.




| x | y |
|---|---|
| $v_x = v_i\cos\theta$ | $v_{iy} = v\sin\theta$ |
| $t = ?$ | $a_y = -9.8\ \frac{m}{s^2}$ |
| $d_x = ?$ | $d_y = 0$ |
| | $t = ?$ |

$$d_y = v_{iy}t + \tfrac{1}{2}a_y t^2$$

$$0 = t\underbrace{\left(v_{iy} + \tfrac{1}{2}a_y t\right)}$$

$$v_{iy} + \tfrac{1}{2}a_y t = 0$$

$$t = -\frac{2v_{iy}}{a_y} = -\frac{2v_i\sin\theta}{a_y}$$

$$d_x = v_x t$$

$$d_x = v_i\cos\theta\left(-\frac{2v_i\sin\theta}{a_y}\right)$$

$$d_x(\theta) = -\frac{2v_i^2}{a_y}\sin\theta\cos\theta$$

$$\frac{d}{d\theta}d_x(\theta) = -\frac{2v_i^2}{a_y}\left(\cos^2\theta - \sin^2\theta\right) = 0$$

$$\frac{d}{d\theta}d_x(\theta) = 0$$

WHEN $d_x$ IS MAX

$$\cos^2\theta - \sin^2\theta = 0$$

$$\cos^2\theta = \sin^2\theta$$

$$1 = \frac{\sin^2\theta}{\cos^2\theta}$$

$$1 = \tan^2\theta$$

$$1 = \tan\theta$$

$$\theta = \tan^{-1}(1) = 45^\circ$$