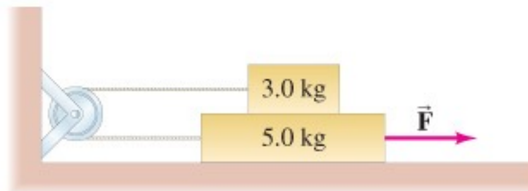
A 3.0 kg block sits on top of a 5.0 kg block which is on a horizontal surface. The 5.0 kg block is pulled to the right with a force ***F*** as shown in the figure. The coefficient of static friction between all surfaces is 0.6 and the kinetic coefficient is 0.4.

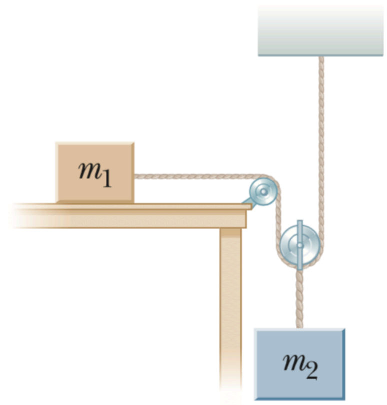


a) What is the minimum value of ***F*** needed to move the two blocks?
b) If the force is 10% greater than your answer for a), what is the acceleration of each block?

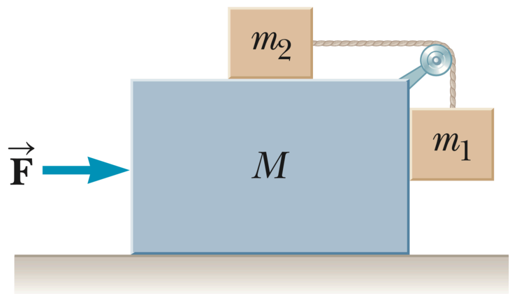
In the system below, the pulleys and the cord are light, all surfaces are frictionless, and the cord does not stretch.

a) How does the acceleration of block 1 compare with the acceleration of block 2?

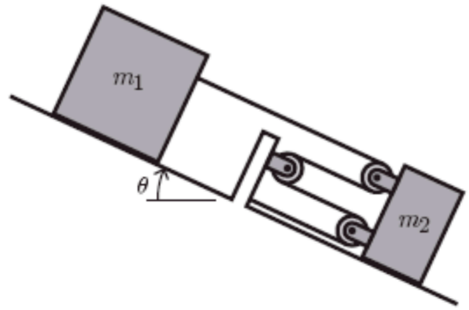
b) What is the acceleration of each block if *m*1 = 2 kg and *m*2 = 3 kg?



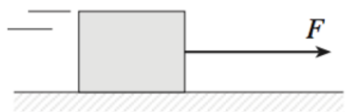
What horizontal force must be applied to a large block of mass M shown so that the blocks remain stationary relative to M? Assume all surfaces and the pulley are frictionless. Notice that the force exerted by the string accelerates $m2$.



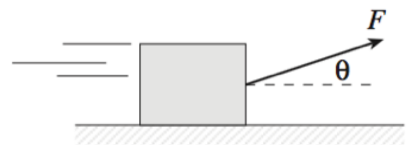
For the pulley system shown, derive an expression for the rope tension in terms of *m*1, *m*2, *g* and *θ*.



A crate is being accelerated across a rough concrete floor by a rope as shown in position 1 below. It is noticed that when the rope is lifted to a small angle *θ* as shown in position 2 the acceleration increases (*F* stays the same).



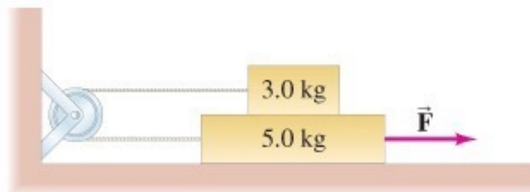
Position 1



Position 2

a) Using principles of physics, explain why this is the case.
b) What *θ* results in the greatest acceleration? Answer in terms of *μ*. *Requires calculus.*

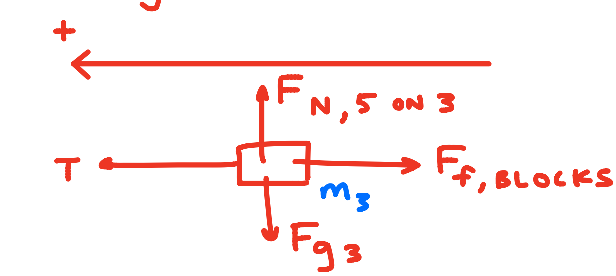
A 3.0 kg block sits on top of a 5.0 kg block which is on a horizontal surface. The 5.0 kg block is pulled to the right with a force ***F*** as shown in the figure. The coefficient of static friction between all surfaces is 0.6 and the kinetic coefficient is 0.4.



a) What is the minimum value of ***F*** needed to move the two blocks?

b) If the force is 10% greater than your answer for a), what is the acceleration of each block?

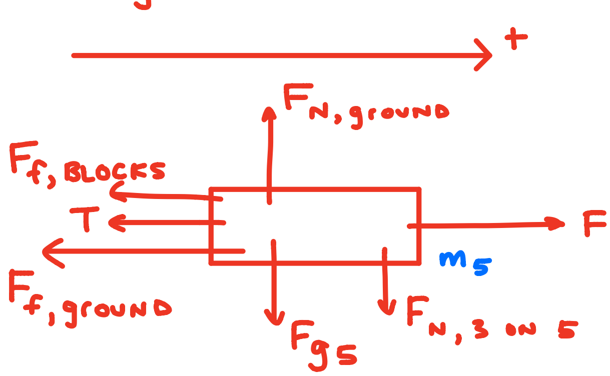
a) 3.0 kg:



$F_{N,5\ on\ 3} = F_{g3}$

$F_{N,5\ on\ 3} = m_3 g$

5.0 kg:



$F_{N,3\ on\ 5} = F_{N,5\ on\ 3}$

$F_{N,3\ on\ 5} = m_3 g$

$F_{N,ground} = F_{g5} + F_{N,3\ on\ 5}$

$F_{N,ground} = m_5 g + m_3 g$

$$F_{NET} = M\cancel{a}^{\,0}$$

$$F - F_{f,ground} - F_{f,\,blocks} \cancel{-T} \cancel{+T} - F_{f,\,blocks} = 0$$

$$F - \mu_s F_{N,ground} - \mu_s F_{N,\,3\text{ on }5} - \mu_s F_{N,\,5\text{ on }3} = 0$$

$$F = \mu_s F_{N,ground} - \mu_s F_{N,\,3\text{ on }5} - \mu_s F_{N,\,5\text{ on }3}$$

$$= \mu_s (m_3 g + m_5 g) - \mu_s m_3 g - \mu_s m_3 g$$

$$= \mu_s g (3m_3 + m_5)$$

$$= (0.6)(9.8)(3(3.0) + 5.0)$$

$$= 82.32 \longrightarrow \boxed{F = 82.3\ N}$$

b) $F = 1.1 \times 82.32$

$F = 90.552\ N$

$$F_{NET} = Ma$$

$$F - F_{f,ground} - F_{f,\,blocks} \cancel{-T} \cancel{+T} - F_{f,\,blocks} = (m_5 + m_3)a$$

$$F - \mu_k F_{N,ground} - \mu_k F_{N,\,3\text{ on }5} - \mu_k F_{N,\,5\text{ on }3} = (m_5 + m_3)a$$

$$F - \mu_k (m_5 g + m_3 g) - \mu_k m_3 g - \mu_k m_3 g = (m_5 + m_3)a$$

$$F - \mu_k g (3m_3 + m_5) = (m_5 + m_3)a$$

$$a = \frac{F - \mu_k g (3m_3 + m_5)}{m_5 + m_3}$$

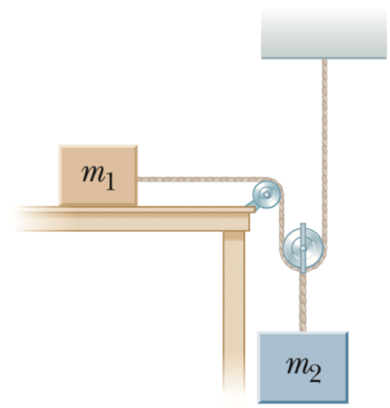
$$= \frac{90.552 - (0.4)(9.8)(3(3.0) + 5.0)}{5.0 + 3.0}$$

$$= 4.459 \longrightarrow \boxed{a = 4.46\ \frac{m}{s^2}}$$

In the system below, the pulleys and the cord are light, all surfaces are frictionless, and the cord does not stretch.

a) How does the acceleration of block 1 compare with the acceleration of block 2?

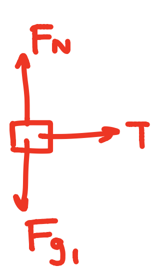
b) What is the acceleration of each block if $m1 = 2$ kg and $m2 = 3$ kg?



a) IF BLOCK 2 MOVES DOWNWARDS BY ONE UNIT, TWO UNITS OF CORD MUST PASS OVER THE PULLEY ATTACHED TO THE TABLE. THIS IS TRUE BECAUSE ONE UNIT OF CORD MUST BE ADDED TO EACH OF THE VERTICAL SECTIONS OF CORD ABOVE BLOCK 2. THUS, BLOCK 2 MUST ALWAYS MOVE AT TWICE THE SPEED OF BLOCK 2.

$v_1 = 2v_2$ AND $a_1 = 2a_2$ AT ALL TIMES.

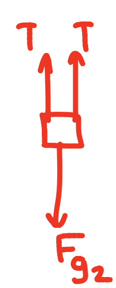
b) BLOCK 1:



$F_{NET} = m_1 a_1$

$T = m_1 a_1$

BLOCK 2:



$F_{NET} = m_2 a_2$

$F_{g_2} - 2T = m_2 a_2$

$m_2 g - 2T = m_2 a_2$

$a_1 = 2a_2$

$$m_2 g - 2m_1 a_1 = m_2 a_2$$

$$m_2 g - 4m_1 a_2 = m_2 a_2$$

$$m_2 g = m_2 a_2 + 4m_1 a_2$$

$$m_2 g = (m_2 + 4m_1) a_2$$

$$a_2 = \frac{m_2 g}{m_2 + 4m_1}$$

$$= \frac{3\,(9.8)}{3 + 4(2)}$$

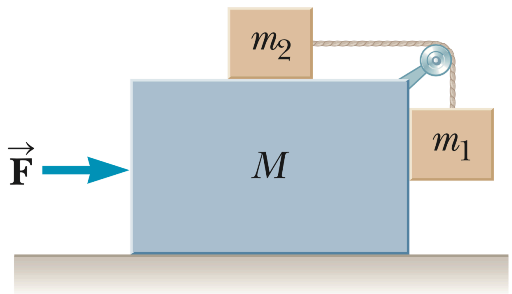
$$= 2.\overline{672} \longrightarrow \boxed{a_2 = 2.67\,\frac{m}{s^2}}$$

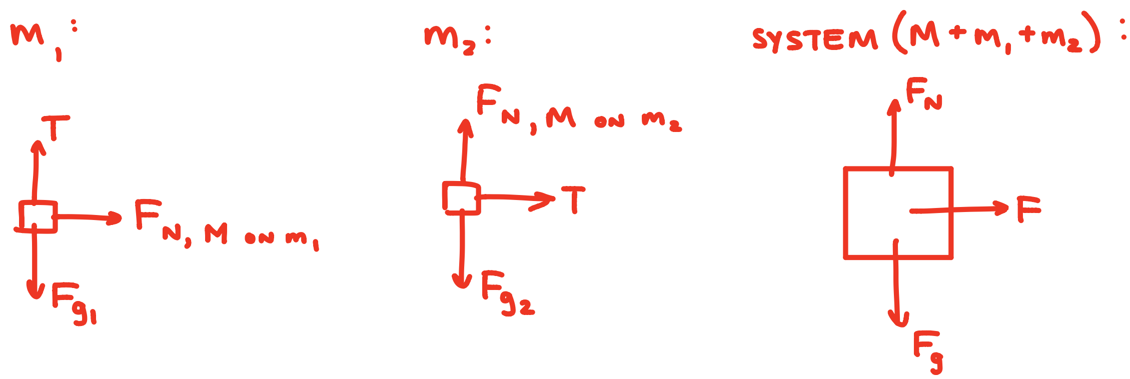
$$a_1 = 2a_2$$

$$= 2(2.\overline{672})$$

$$= 5.3\overline{45} \longrightarrow \boxed{a_1 = 5.35\,\frac{m}{s^2}}$$

What horizontal force must be applied to a large block of mass *M* shown so that the blocks remain stationary relative to *M*? Assume all surfaces and the pulley are frictionless. Notice that the force exerted by the string accelerates *m*2.





FOR m_1 TO REMAIN STATIONARY RELATIVE TO M, m_1 MUST NOT ACCELERATE VERTICALLY.

$$T = F_{g_1}$$

$$T = m_1 g$$

THE FORCE OF TENSION ACCELERATES m_2.

$$F_{NET} = m_2 a$$

$$T = m_2 a$$

$$m_1 g = m_2 a$$

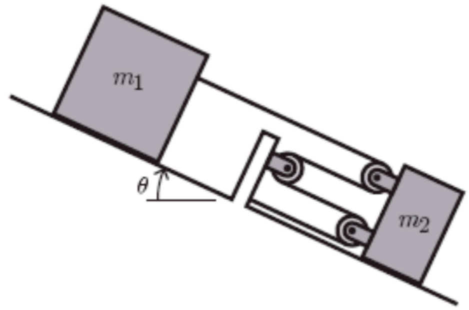
$$a = \frac{m_1}{m_2} g$$

FOR m_1 TO REMAIN STATIONARY RELATIVE TO M, M MUST ACCELERATE AT THE SAME RATE. HENCE, TO KEEP ALL BLOCKS STATIONARY RELATIVE TO EACH OTHER, THE FORCE F MUST ACCELERATE THE ENTIRE SYSTEM TOWARD THE RIGHT AT THIS RATE.

$$F_{NET} = Ma$$

$$F = (M + m_1 + m_2)\left(\frac{m_1}{m_2} g\right)$$

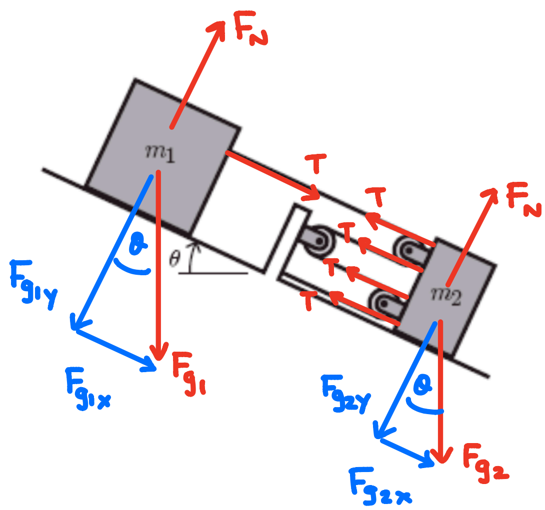
For the pulley system shown, derive an expression for the rope tension in terms of $m1$, $m2$, g and θ.



Solution

Part 1: https://www.youtube.com/watch?v=8P2eV4-s9AI

Part 2: https://www.youtube.com/watch?v=Gr8stm279_k



$$F_{g1x} = m_1 g \sin\theta$$

$$F_{g2x} = m_2 g \sin\theta$$

$$a_1 = 4a_2$$

m_1:

$$F_{NET} = m_1 a_1$$

$$F_{g1x} + T = m_1 a_1$$

$$m_1 g \sin\theta + T = m_1 a_1$$

$$m_1 g \sin\theta + T = 4 m_1 a_2$$

$$a_2 = \frac{m_1 g \sin\theta + T}{4 m_1}$$

m_2:

$$F_{NET} = m_2 a_2$$

$$F_{g2x} - 4T = m_2 a_2$$

$$m_2 g \sin\theta - 4T = m_2 a_2$$

$$a_2 = \frac{m_2 g \sin\theta - 4T}{m_2}$$

$$\frac{m_1 g \sin\theta + T}{4m_1} = \frac{m_2 g \sin\theta - 4T}{m_2}$$

$$m_2(m_1 g \sin\theta + T) = 4m_1(m_2 g \sin\theta - 4T)$$

$$m_1 m_2 g \sin\theta + m_2 T = 4 m_1 m_2 g \sin\theta - 16 m_1 T$$

$$16 m_1 T + m_2 T = 4 m_1 m_2 g \sin\theta - m_1 m_2 g \sin\theta$$

$$T(16 m_1 + m_2) = 3 m_1 m_2 g \sin\theta$$

$$T = \frac{3 m_1 m_2 g \sin\theta}{16 m_1 + m_2}$$

A crate is being accelerated across a rough concrete floor by a rope as shown in position 1 below. It is noticed that when the rope is lifted to a small angle *θ* as shown in position 2 the acceleration increases (*F* stays the same).

Position 1 Position 2

a) Using principles of physics, explain why this is the case.

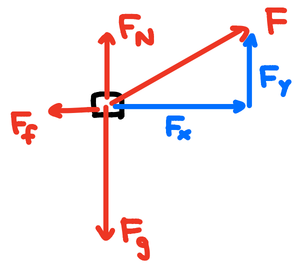
b)What *θ* results in the greatest acceleration? Answer in terms of *μ*. *Requires calculus.*

a) ACCORDING TO NEWTON'S SECOND LAW, ACCELERATION IS DIRECTLY PROPORTIONAL TO THE NET FORCE AND INVERSELY PROPORTIONAL TO THE MASS.

BY LIFTING THE ROPE, THE FORCE F IN POSITION 2 HAS A VERTICAL COMPONENT WHICH BALANCES A PORTION OF F_g, LEAVING F_N AND THEREFORE F_f REDUCED.

AS THE HORIZONTAL COMPONENT OF F HAS CHANGED VERY LITTLE FROM POSITION 1 TO POSITION 2, F_{NET} AND THEREFORE ACCELERATION HAVE BEEN INCREASED.

b)



$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

$$F_N + F_y = F_g$$

$$F_N = F_g - F_y$$

$$F_N = mg - F\sin\theta$$

$$F_{NET} = ma$$

$$F_x - F_f = ma$$

$$F\cos\theta - \mu F_N = ma$$

$$F\cos\theta - \mu(mg - F\sin\theta) = ma$$

$$F\cos\theta - \mu mg + \mu F\sin\theta = ma$$

$$a = \frac{F}{m}\cos\theta + \frac{\mu F}{m}\sin\theta - \mu mg$$

$$\frac{da}{d\theta} = -\frac{F}{m}\sin\theta + \frac{\mu F}{m}\cos\theta = 0$$

$$\frac{\mu \cancel{F}}{\cancel{m}}\cos\theta = \frac{\cancel{F}}{\cancel{m}}\sin\theta$$

$$\mu = \tan\theta$$

$$\theta = \tan^{-1}(\mu)$$