

Relationships Between Variables - Solutions

1. For each pair of variables, write the equation (rearrange the equation if necessary) and determine the relationship. The relationships should be a description of how the first variable depends on the second.

a) $A = \pi r^2$

$$A \propto r^2$$

Area is directly proportional to the square of radius.

b) $V = \frac{4}{3}\pi r^3$

$$V \propto r^3$$

Volume is directly proportional to the cube of radius.

c) $d = vt$

$$d \propto v$$

Displacement is directly proportional to velocity.

d) $d = \frac{1}{2}at^2$

$$d \propto t^2$$

Displacement is directly proportional to the square of time.

e) $a = \frac{F_{\text{NET}}}{m}$

$$a \propto F_{\text{NET}}$$

Acceleration is directly proportional to net force.

f) $a = \frac{F_{\text{NET}}}{m}$

$$a \propto \frac{1}{m}$$

Acceleration is inversely proportional to mass.

g) $F_f = \mu F_N$

$$F_f \propto F_N$$

Force of friction is directly proportional to the normal force.

h) $\Delta x = \frac{F_s}{k}$

$$\Delta x \propto \frac{1}{k}$$

Displacement from equilibrium position is inversely proportional to the spring constant.

i) $g = G \frac{M}{r^2}$

$$g \propto M$$

Gravitational field strength is directly proportional to the mass.

j) $F_g = G \frac{m_1 m_2}{r^2}$

$$F_g \propto \frac{1}{r^2}$$

Gravitational force is inversely proportional to the square of separation distance.

k) $p = mv$

$$p \propto v$$

Momentum is directly proportional to velocity.

l) $E_p = mgh$

$$E_p \propto h$$

Gravitational potential energy is directly proportional to height.

m) $E_k = \frac{1}{2}mv^2$

$$E_k \propto m$$

Kinetic energy is directly proportional to mass.

n) $E_k = \frac{1}{2}mv^2$

$$E_k \propto v^2$$

Kinetic energy is directly proportional to the square of velocity.

o) $P = \frac{W}{t}$

$$P \propto \frac{1}{t}$$

Power is inversely proportional to time.

p) $I = \frac{V}{R}$

$$I \propto \frac{1}{R}$$

Current is inversely proportional to resistance.

2. Determine how the following variables change.

- a) 4x
- b) 1/8x
- c) 5x
- d) 1/16x
- e) 3x
- f) 1/2x
- g) 1/3x
- h) 2x
- i) 4x
- j) 1/36x
- k) 2.5x
- l) 1/10x
- m) 1/2x
- n) 1/4x
- o) 1/2x
- p) 5/4x

3. Consider the equation for kinetic energy

$$E_k = \frac{1}{2}mv^2$$

where m represents the mass and v represents the velocity.

Determine the change in the kinetic energy for each of the following changes.

- a) 2x
- b) 4x
- c) 1/2x
- d) 1/4x
- e) 27x
- f) 1/27x
- g) 1/2x
- h) 1x (no change)

4. Consider the equation for resistance

$$R = \frac{\rho l}{A}$$

where ρ represents the resistivity, l represents the length of the wire, and A represents the cross-sectional area.

Determine the change in the resistance for each of the following changes.

- a) 3x
- b) 2x
- c) 1/4x
- d) 1/4x
- e) 1x (no change)
- f) 3/2x
- g) 1/12x
- h) 2/3x

5. Consider the equation for the gravitational force between two masses

$$F_g = G \frac{m_1 m_2}{r^2}$$

where G represents the gravitation constant, m_1 and m_2 represent the masses and r represents the separation distance.

Two objects are separated by a distance of 1000 km (from their centres). The gravitational force at this distance is 500 N. Determine the gravitational force between the masses for the following changes.

- a) 1000 N
- b) 4500 N
- c) 20 N
- d) 8000 N
- e) 320 N
- f) 625 N
- g) 500 N
- h) 1172 N