Physics 11
M. Lam

Name:

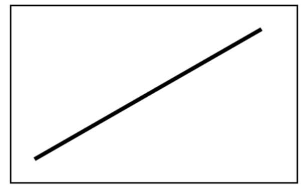
Block:

Relationships Between Variables

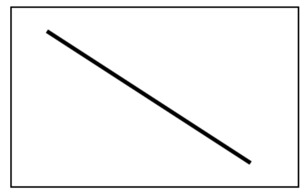
A relationship in physics describes how a change to one variable will affect another.

There are two types of relationships:

Direct: The two variables do the <u>same thing</u>. If one variable increases, so does the other and vice versa.



Inverse: The two variables do the <u>opposite</u>. If one variable increases, the other decreases.



Relationships are usually written as proportionalities and use the symbol $\propto$ to symbolize the proportion. To make an equation we add a constant (usually *k*, if unknown) and change the proportionality sign into an equals sign.

Note:
$\propto$ means "proportional to"
k is the constant of proportionality

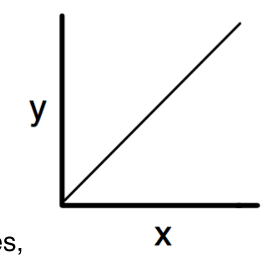
Directly Proportional (Linear)

We say: *y* is directly proportional to *x*

Proportionality: $y \propto x$

Equation: $y = kx$

Meaning: If *x* increases, *y* increases proportional to *x*. For example, if *x* doubles, *y* doubles; if *x* is halved, *y* is halved.



<u>Example: What will be the change in the force of friction if the normal force is decreased by a factor of three?</u>

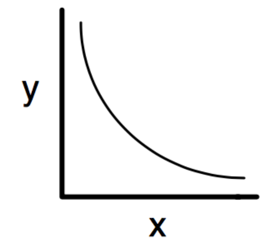
$$F_f = \mu F_N$$

Force of friction is directly proportional to normal force.

If the normal force decreases, so does the force of friction.

If F_N is decreased by a factor of three, F_f is decreased by a factor of three (×1/3).

Inversely Proportional



We say: *y* is inversely proportional to *x* (or *y* is directly proportional to 1/*x*)

Proportionality: $y \propto \frac{1}{x}$

Equation: $y = \frac{k}{x}$

Meaning: If *x* increases, *y* decreases proportional to *x*. For example, if *x* doubles, *y* is halved; if *x* decreases by a factor of three (ie. one third of its original value), *y* is tripled.

Example: What will be the change in the acceleration if the mass is increased by a factor of five?

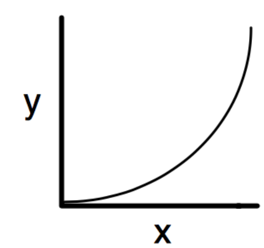
$$a = \frac{F_{NET}}{m}$$

Acceleration is inversely proportional to mass.

If the mass is increased (and the net force is kept constant), then the acceleration decreases.

If *m* is increased by a factor of five, the acceleration is decreased by a factor of five (×1/5).

Direct Square (Quadratic)



We say: *y* is directly proportional to the square of *x*

Proportionality: $y \propto x^2$

Equation: $y = kx^2$

Meaning: If *x* increases, *y* increases proportional to x^2. For example, if *x* doubles, *y* quadruples; if *x* decreases by a factor of 3; *y* decreases by a factor of 9.

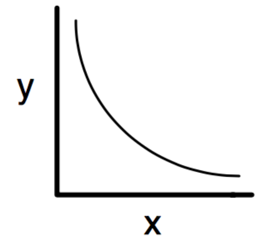
Example: What will be the change in the kinetic energy if velocity is tripled?

$$E_k = \frac{1}{2}mv^2$$

The kinetic energy is directly proportional to the square of velocity.

If *v* is tripled, E_k is increased by a factor of nine (×9).

Inverse Square



We say: *y* is inversely proportional to the square of *x*

Proportionality: $y \propto \frac{1}{x^2}$

Equation: $y = \frac{k}{x^2}$

Meaning: If *x* increases, *y* decreases proportional to x^2. For example, if *x* doubles, *y* decreases by a factor of four; if *x* decreases by a factor of three, *y* increases by a factor of nine.

Example: What will be the change in the gravitational force if the separation distance is increased by a factor of 10?

$$F_g = G\frac{m_1 m_2}{r^2}$$

Gravitational force is inversely proportional to the square of the separation distance.

If *r* is increased by a factor of ten, F_g is decreased by a factor of 100 (×1/100).

Relationships between multiple variables

If there is a relationship between multiple variables, all of which change, you can look at each variable independently and then find their combined effect.

Example: For an object in uniform motion, what will be the change in the displacement if the velocity is increased by a factor of four but the time is halved?

$$d = vt$$

Displacement is directly proportional to both velocity and time.

If v is increased by a factor of four, d is increased by a factor of four (×4)
If t is halved, d is halved (×1/2)

(×4)(×1/2) = ×2

The displacement will be doubled.

Example: What will be the change in the gravitational field if the mass is doubled and the radius is tripled?

$$g = G\frac{M}{r^2}$$

Gravitational field strength is directly proportional to the mass and inversely proportional to the square of the distance.

If m is doubled, g is doubled (×2).
If r is tripled, g is decreased by a factor of nine (×1/9).

(×2)(×1/9) = ×2/9

The gravitational field will be 2/9 of its original value.