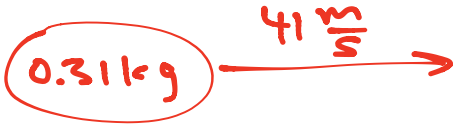
# MOMENTUM (2-DIMENSIONS) - SOLUTIONS

1. INITIAL

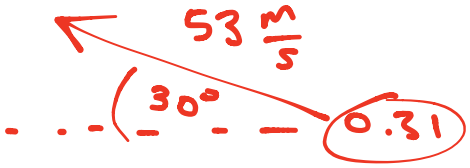



$p_i = mv_i$

$= (0.31)(41)$

$= 12.71 \text{ kg} \frac{m}{s}$

FINAL



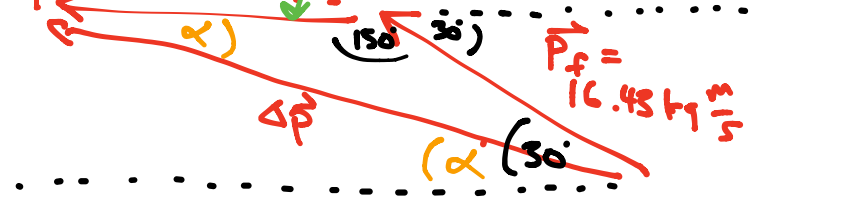

$p_f = mv_f$

$= (0.31)(53)$

$= 16.43 \text{ kg} \frac{m}{s}$

IMPULSE: $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

note that we use the opposite direction of $\vec{p}_i$ in the vector diagram




magnitude:

$(\Delta p)^2 = p_i^2 + p_f^2 - 2(p_i)(p_f)\cos 150°$

$= (12.71)^2 + (16.43)^2 - 2(12.71)(16.43)\cos 150°$

$= 793.19$

$\Delta p = \sqrt{793.19}$

$= 28.16$

DIRECTION:

$$\frac{\Delta P}{\sin 150^\circ} = \frac{P_2}{\sin \alpha}$$

$$\sin \alpha = \frac{P_2}{\Delta P} \sin 150^\circ$$

$$\alpha = \sin^{-1}\left(\frac{P_2}{\Delta P} \sin 150^\circ\right)$$

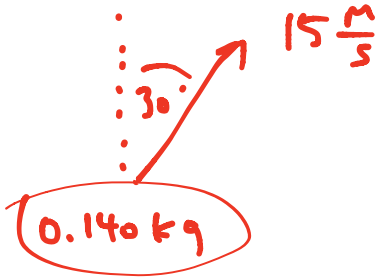
$$= \sin^{-1}\left(\frac{16.43}{28.16} \sin 150^\circ\right)$$

$$= 16.96$$

28 Ns 17° N of W

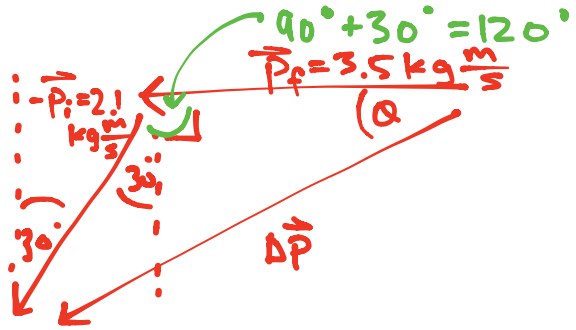
(UPWARD ANGLE TOWARDS THE PITCHER)

2. INITIAL




FINAL


IMPULSE $= \Delta\vec{p} = \vec{p}_f - \vec{p}_i$




$p_i = mv_i$

$= (0.140)(15)$

$= 2.1\ kg\frac{m}{s}$

$p_f = mv_f$

$= (0.140)(25)$

$= 3.5\ kg\frac{m}{s}$

magnitude:

$(\Delta p)^2 = p_i^2 + p_f^2 - 2(p_i)(p_f)\cos 120°$

$= (2.1)^2 + (3.5)^2 - 2(2.1)(3.5)\cos 120°$

$= 24.01$

$\Delta p = \sqrt{24.01}$

$\approx 4.9\ Ns$

DIRECTION:

$$\frac{\Delta p}{\sin 120} = \frac{p_i}{\sin\theta}$$

$$\sin\theta = \frac{p_i}{\Delta p}\sin 120°$$

$$\theta = \sin^{-1}\left(\frac{p_i}{\Delta p}\sin 120°\right)$$

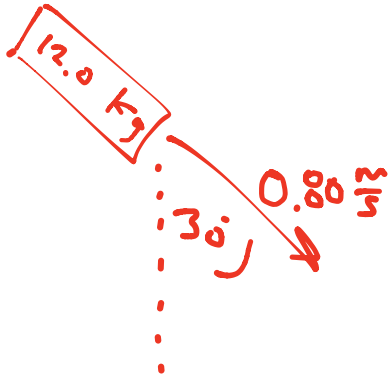
$$= \sin^{-1}\left(\frac{2.1}{4.9}\sin 120°\right)$$

$$= 21.79°$$

4.9 Ns 22° SofW

3. INTIAL


FINAL




$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

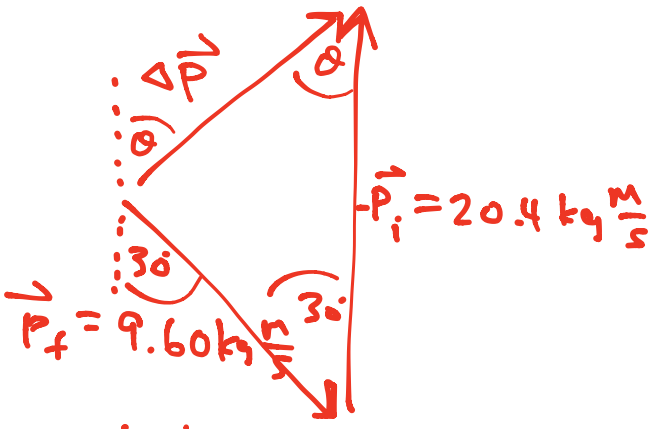
$$p_i = mv_i$$

$$= (12.0)(1.70)$$

$$= 20.4\ \text{kg}\frac{\text{m}}{\text{s}}$$

$$p_f = mv_f$$

$$= 9.60\ \text{kg}\frac{\text{m}}{\text{s}}$$




magnitude:

$$\Delta p^2 = p_i^2 + p_f^2 - 2(p_i)(p_f)\cos 30°$$

$$= (20.4)^2 + (9.60)^2 - 2(20.4)(9.60)\cos 30°$$

$$= 169.12$$

$$\Delta p = \sqrt{169.12}$$

$$= 13.00\ kg\frac{m}{s}$$

DIRECTION:

$$\frac{\Delta p}{\sin 30} = \frac{p_f}{\sin\theta}$$

$$\sin\theta = \frac{p_f}{\Delta p}\sin 30$$

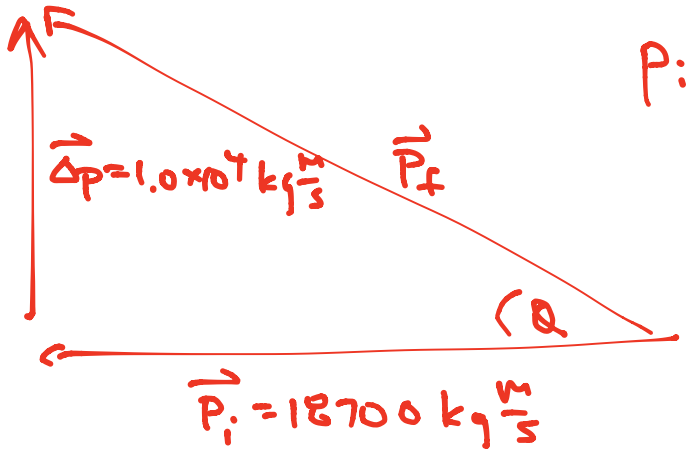
$$\theta = \sin^{-1}\left(\frac{p_f}{\Delta p}\sin 30\right)$$

$$= \sin^{-1}\left(\frac{9.60}{13.00}\sin 30\right)$$

$$= 21.66°$$

$13\ kg\frac{m}{s}$ 22° E OF N

4. $\vec{P_f} = \vec{P_i} + \Delta\vec{P}$




$P_i = m v_i$

$= (1100)(17)$

$= 18700 \ kg\frac{m}{s}$

magnitude:

$P_f^2 = P_i^2 + (\Delta p)^2$

$P_f = \sqrt{P_i^2 + (\Delta p)^2}$

$= \sqrt{(18700)^2 + (1.0 \times 10^4)^2}$

$= 21205 \ kg\frac{m}{s}$

DIRECTION:

$\tan\theta = \frac{\Delta p}{P_i}$

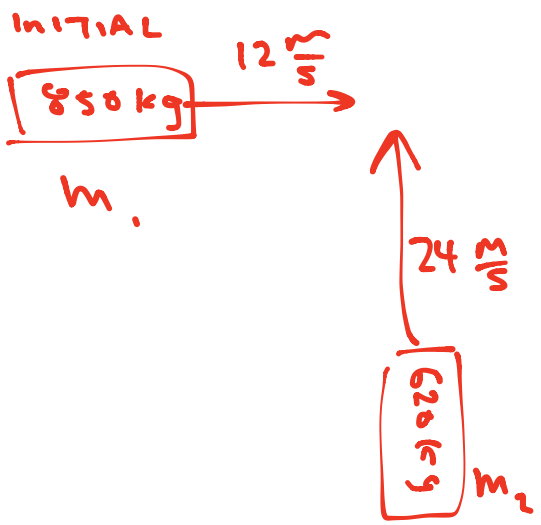
$\theta = \tan^{-1}\left(\frac{\Delta p}{P_i}\right)$

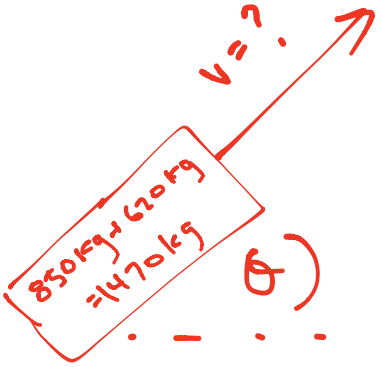
$= \tan^{-1}\left(\frac{1.0 \times 10^4}{18700}\right)$

$= 28.14^\circ$

$28^\circ$ N of W

5.




$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_f$$

$p_1 = m_1 v_{1i}$

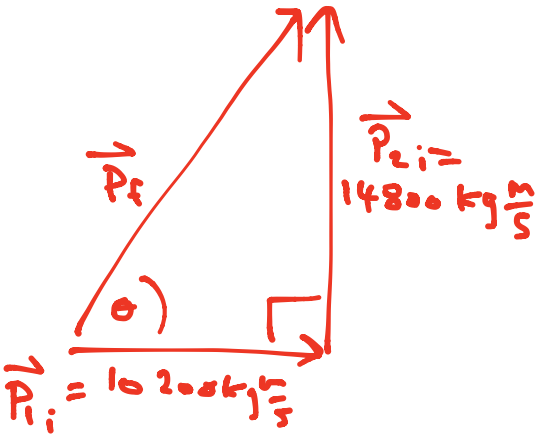
$= (850)(12)$

$= 10200 \text{ kg} \frac{\text{m}}{\text{s}}$

$p_2 = m_2 v_{2i}$

$= (620)(24)$

$= 14880 \text{ kg} \frac{\text{m}}{\text{s}}$



MAGNITUDE:

$$P_f^2 = P_{1i}^2 + P_{2i}^2$$

$$P_f = \sqrt{P_{1i}^2 + P_{2i}^2}$$

$$P_f = \sqrt{(10200)^2 + (14880)^2}$$

$$= 18040 \text{ kg}\frac{\text{m}}{\text{s}}$$

↓ velocity

$$P_f = M v_f$$

$$v_f = \frac{P_f}{M}$$

$$= \frac{18040}{1470}$$

$$= 12.27 \frac{\text{m}}{\text{s}}$$

DIRECTION

$$\tan\theta = \frac{P_{2i}}{P_{1i}}$$

$$\theta = \tan^{-1}\left(\frac{P_{2i}}{P_{1i}}\right)$$

$$= \tan^{-1}\left(\frac{14880}{10200}\right)$$

$$= 56^\circ$$

↓

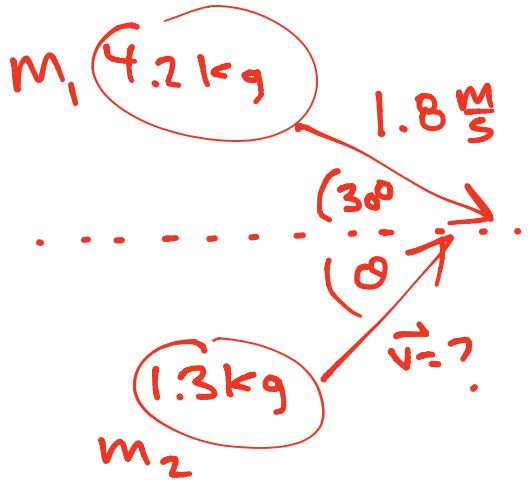
→

$12 \frac{\text{m}}{\text{s}}$ $56^\circ$ N of E

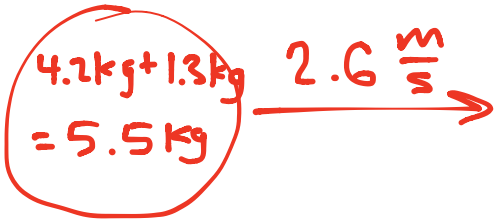
OR

$12 \frac{\text{m}}{\text{s}}$ $34^\circ$ E of N

6. INITIAL:



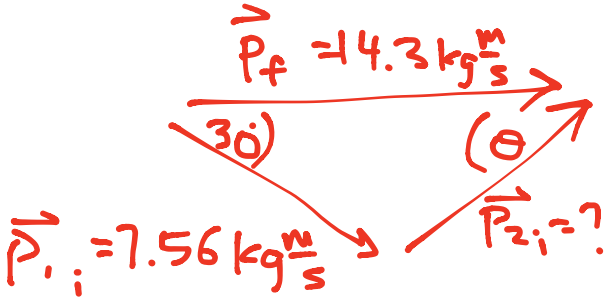

FINAL




$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_f$$

$$p_{1i} = m_1 v_{1i}$$
$$= (4.2)(1.8)$$
$$= 7.56 \text{ kg} \frac{m}{s}$$




$$p_f = M v_f$$
$$= (5.5)(2.6)$$
$$= 14.3 \text{ kg} \frac{m}{s}$$

Magnitude:

$$p_{2i}^2 = p_{1i}^2 + p_f^2 - 2(p_{1i})(p_f)\cos 30°$$
$$= (7.56)^2 + (14.3)^2 - 2(7.56)(14.3)\cos 30°$$
$$= 74.40$$

$$P_{2i} = \sqrt{74.40}$$

$$= 8.63 \ kg\frac{m}{s}$$

DIRECTION:

$$\frac{P_{2i}}{\sin 30^\circ} = \frac{P_{1i}}{\sin\theta}$$

$$\sin\theta = \frac{P_{1i}}{P_{2i}} \sin 30^\circ$$

$$\theta = \sin^{-1}\left(\frac{P_{1i}}{P_{2i}} \sin 30^\circ\right)$$

$$= \sin^{-1}\left(\frac{7.56}{8.63} \sin 30\right)$$

$$= 25.98^\circ$$

$$P_{2i} = m_2 v_{2i}$$

$$v_{2i} = \frac{P_{1i}}{m_2}$$

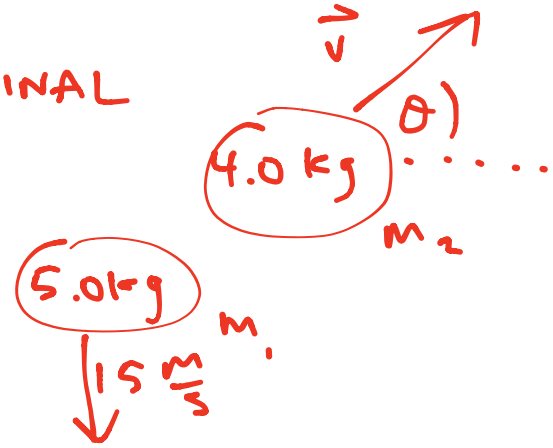
$$= \frac{8.63}{1.3}$$

$$= 6.63\frac{m}{s}$$

$6.6\frac{m}{s}$ 26° N of E

7. INITIAL

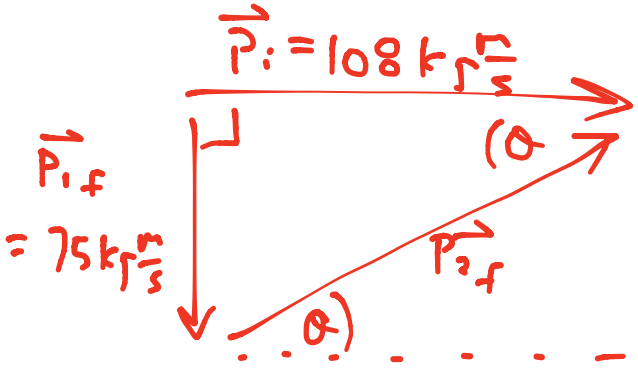

FINAL




$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$\vec{p}_i = \vec{p}_{1f} + \vec{p}_{2f}$$

$$p_i = M v_i$$
$$= (9.0)(12)$$
$$= 108 \text{ kg} \frac{m}{s}$$




$$p_{1f} = m_1 v_{1f}$$
$$= (5.0)(15)$$
$$= 75 \text{ kg} \frac{m}{s}$$

Magnitude:

$$p_{2f}^2 = p_i^2 + p_f^2$$

$$p_{2f} = \sqrt{p_i^2 + p_f^2}$$
$$= \sqrt{(108)^2 + (75)^2}$$
$$= 131.49 \text{ kg} \frac{m}{s}$$

DIRECTION

$$\tan\theta = \frac{p_{1f}}{p_i}$$

$$\theta = \tan^{-1}\left(\frac{p_{1f}}{p_i}\right)$$
$$= \tan^{-1}\left(\frac{75}{108}\right)$$
$$= 34.78^\circ$$


$$P_{2f} = m_2 v_{2f}$$

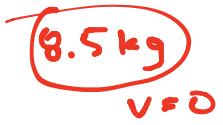
$$v_{2f} = \frac{P_{2f}}{m_2}$$

$$= \frac{131.49}{4.0}$$

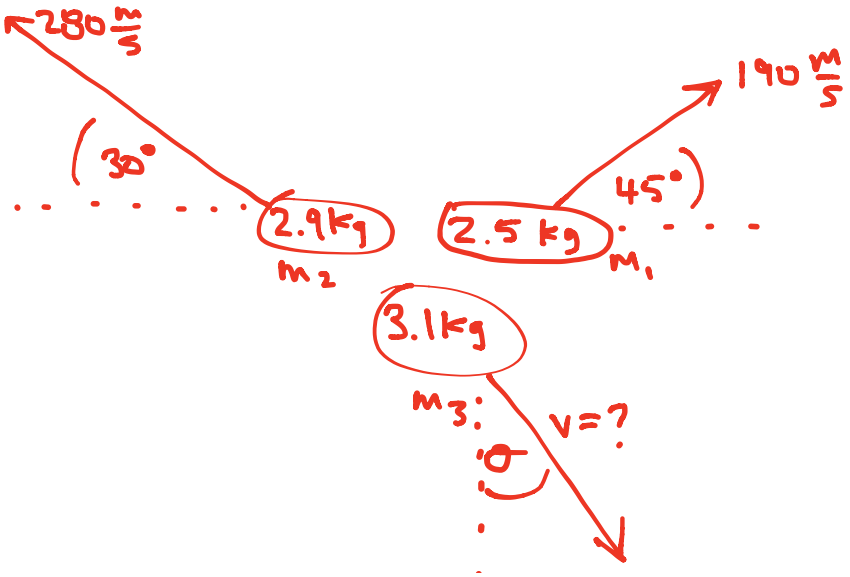
$$= 32.87 \frac{m}{s}$$

$33 \frac{m}{s}$ N of E

8. INITIAL




FINAL

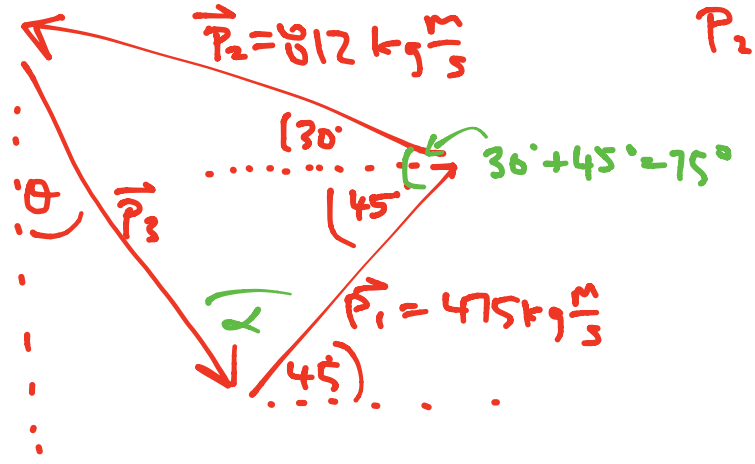


$$\sum \vec{p}_i = \sum \vec{p}$$

$$0 = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$$

$$p_1 = m_1 v_1$$
$$= (25)(190)$$
$$= 475 \text{ kg} \frac{\text{m}}{\text{s}}$$

$$p_2 = m_2 v_2$$
$$= (2.9)(280)$$
$$= 812 \text{ kg} \frac{\text{m}}{\text{s}}$$




magnitude:

$$p_3^2 = p_1^2 + p_2^2 - 2(p_1)(p_2)\cos 75°$$
$$= (475)^2 + (812)^2 - 2(475)(812)\cos 75°$$
$$= 685316$$
$$p_3 = \sqrt{685316}$$
$$= 827.8 \text{ kg} \frac{\text{m}}{\text{s}}$$

DIRECTION:

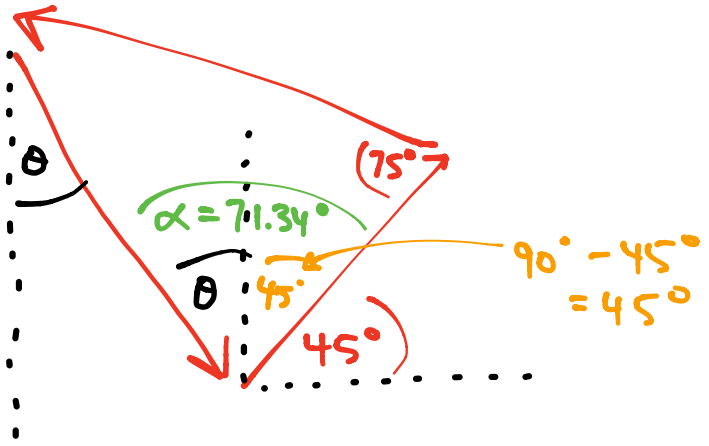
$$\frac{P_3}{\sin 75^\circ} = \frac{P_2}{\sin \alpha}$$

$$\sin \alpha = \frac{P_2}{P_3} \sin 75^\circ$$

$$\alpha = \sin^{-1}\left(\frac{P_2}{P_3} \sin 75^\circ\right)$$

$$= \sin^{-1}\left(\frac{812}{827.8} \sin 75^\circ\right)$$

$$= 71.34^\circ$$




$$\theta = \alpha - 45^\circ$$

$$= 71.34 - 45$$

$$= 26.34^\circ$$

$$P_3 = m_3 v_3$$

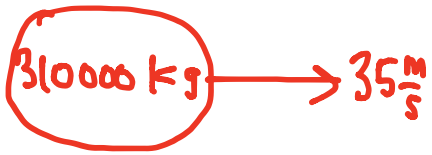
$$v_3 = \frac{P_3}{m_3}$$

$$= \frac{827.8}{3.1}$$

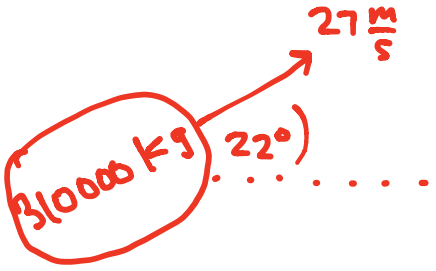
$$= 267 \frac{m}{s}$$

$270 \frac{m}{s}$ 26° E of S

9.a) INITIAL




FINAL




$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$p_i = mv_i$

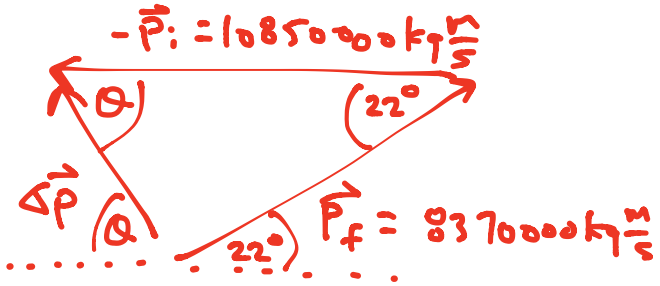
$= (310000)(35)$

$= 10850000 \text{ kg}\frac{\text{m}}{\text{s}}$

$p_f = mv_f$

$= (310000)(27)$

$= 8370000 \text{ kg}\frac{\text{m}}{\text{s}}$




Magnitude:

$(\Delta p)^2 = p_i^2 + p_f^2 - 2(p_i)(p_f)\cos 22°$

$= (10850000)^2 + (8370000)^2 - 2(10850000)(8370000)\cos 22°$

$= 1.9376 \times 10^{13}$

$\Delta p = \sqrt{1.9376 \times 10^{13}}$

$= 4401807 \text{ Ns}$

DIRECTION:

$$\frac{\Delta p}{\sin 22} = \frac{P_f}{\sin\theta}$$

$$\sin\theta = \frac{P_f}{\Delta p}\sin 22^\circ$$

$$\theta = \sin^{-1}\left(\frac{P_f}{\Delta p}\sin 22^\circ\right)$$

$$= \sin^{-1}\left(\frac{8370000}{44701809}\sin 22^\circ\right)$$

$$= 45.42^\circ$$

$4.4 \times 10^6$ N·s $45^\circ$ N of E

b) $\Delta p = F_{NET}\,\Delta t$

$$F_{NET} = \frac{\Delta p}{\Delta t}$$

$$= \frac{4.4 \times 10^6}{45}$$

$$= 97818 \text{ N}$$

98000 N