4.1 Word Problems

Exponential functions can be used to model many real world phenomena, such as population growth, the growth or decay of various substances and the value of investments that are earning compound interest.

Growth and decay: A=$A_0 x^{\frac{t}{T}}$

A- final amount

A_0- initial amount

x- growth or decay value, e.g. half-life use x= $\frac{1}{2}$

double x=2

increase by 20% x=1.2

decrease by 20% x=0.8

t- total time that item is left

T- time of growth or decay, e.g., half-life

Continuous Growth and Decay: A=$A_0 e^{kt}$

A- final amount

A_0- initial amount

e- constant ≈ 2.71828

k- proportional constant

t- time

Compound Interest Formula

Interest calculated on principle plus previously earned interest is called compound interest. If P dollars is deposited at an annually rate *r*, compounded *n* times per year then the compounded amount A is given by the formula:

A=P $(1+\frac{r}{n})^{nt}$

A= final amount

n= number of times yearly interest is compounded in a year

P= principal, or initial amount

t= times in years

R= rate of yearly interest

Earthquakes

Earthquakes are measured on the Richter scale. Each 1 unit on this scale is equivalent to a 10-fold increase in the intensity.

Same for pH (measuring acidity).

Decibels (measuring loudness of sounds) increase of 10 db is 10 times louder

Example 1

A certain culture of bacteria, given suitable conditions, triples every 25h. The initial count of a sample shows 1000 bacteria present.

a) Write an exponential function that models the given conditions.

b) Approximately how many bacteria will there be in 4 days.

c) How many bacteria were there 3 days prior to the count?

Example 2

The intensity of the light below the surface of a particular lake is reduced by 4% for every metre below the surface.

a) Write an exponential function that models the intensity of the light at any depth below the surface.

b) What percent of the original intensity of the light remains 10 m below the surface?

Example3

The half-life of sodium-24 is 14.9 hours. A hospital buys 40-mg sample of sodium-24.

a) How many grams, to the nearest tenth, of sodium-24 will remain after 48 hours?

b) After how long will only 2.5 mg remain?

Example 4

How many more times more powerful is an earthquake of 8.2 on the Richter scale than a 6.5 magnitude earthquake?

Example 5

The $\frac{1}{2}$ life of cobalt-60 is 5.3 years. What percent will remain of a sample after 26.5 years?

Example 6

The $\frac{1}{2}$ life of plutonium-239 is about 25000 years. How much of a given sample will remain after 3000 years?

Example 7

The number of rabbits increases by 20% every 5 days. If there were 258 rabbits after 26 days, how many were there initially?

Example 8

Find the value of an initial investment of $ 2000, after 3 years, if the interest rate is 4.2% per annum compounded quarterly.

Example 9

$15000 is invested in an account paying 3.5%/a. How much will be in the account at the end of 25 years if the interest is compounded

a. Monthly?

b. Weekly?

c. Daily?