Rational Functions

A rational function has the form $f(x)$ = $\frac{g(x)}{h(x)}$, where $g(x)\ and\ h(x)$ are polynomials and $h(x)$ ≠0.

$f(x)=\frac{a_m\,x^m+a_{m-1}\,x^{m-1}+\cdots+a_1\,x+a_0}{b_n\,x^n+b_{n-1}\,x^{n-1}+\cdots+b_1\,x+b_0}$, where $a_m\neq 0$ and $b_m\neq 0$

Examples :

Since the rational functions are expressed as fractions, the denominator cannot equal zero. The values that make the denominator zero are called **non-permissible values**.

If a rational function has non-permissible values, than the graph of the function will not be continuous. The graph will have an **infinite discontinuity** (vertical asymptote on the graph) or **point discontinuity** (hole on the graph).

Note that not all rational functions have non-permissible values! Example: f(x) = $\frac{2x}{x^2+4}$

The **vertical asymptotes** are located at the zeros of the denominator of a rational function (non permissible values)

Let f(x) = $\frac{g(x)}{h(x)}$ be a rational function. If c is a zero of h(x), then the line x=c is a vertical asymptote of the graph of f(x).

Examples:

1) $f(x)=\frac{2-x}{x^2-9}$

2) $f(x)=\frac{x-1}{3x+5}$

3) $g(x)=\frac{2x}{x^2+9}$

4) $h(x)=\frac{2}{x^3+9x}$

Rational functions might have **horizontal asymptotes.**

Let f(x) = $\frac{a_m x^m + a_{m-1} x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0}$, where $a_m \neq 0$ and $b_m \neq 0$

a) If m$< n$, then the line y =0 is a horizontal asymptote
b) If m=n, then the line y =$\frac{a_m}{b_n}$ (ratio of the leading coefficients) is a horizontal asymptote.
c) If m$> n$, then there is no horizontal asymptote.

Examples:

1) $f(x) = \frac{2x}{x^2+4}$

2) $f(x) = \frac{2x^2}{x^2-5}$

3) $f(x) = \frac{x^2-2x+1}{-3x}$

4) Determine the y intercept, x-intercept, the domain, and the equations of the asymptotes for the following rational functions:

a) $f(x) = \frac{x^2-9}{x^2-x-2}$

b) $f(x) = \frac{1}{x+2} - \frac{1}{x-2} + 1$

d) $f(x) = \frac{2x^2-18}{x^3-9x}$