1.3 Operations with Functions. Composition.

Must know from Math 10:

If $f(x) = 2x + 1$ If $g(x) = -x^2 + x - 1$

Determine: Determine:

$f(a) =$ $g(a) =$

$f(3x) =$ $g(a - 1) =$

$f(x^2 - 1) =$

It is possible to combine functions by a method called **composition**.

<u>Example #1</u>: If $f(x) = x - 1$ and $g(x) = x + 2$, determine the composition of f and g at point $x = 3$.

Visual Representation: Algebra:

<u>Example #2</u>: If $f(x) = x^2 - 1$ and $g(x) = x - 2$, determine the composition of f and g at point $x = -1$.

Visual Representation: Algebra:

For two functions $f \; and \; g$ both defined on the real numbers, the composition is defined as

The _____ of function f must include the range of g.

Visual representation:

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1) Evaluate the _____ function first. Replace $g(x)$ with the formula for g.
2) Use the _____ from the inner function as the input for the _____ function f(x).
3) Write the answer in simplest form.
4) **Remember!**

It is possible to do the composite of a function with itself: fof.

Do not confuse composition of function with multiplication! ____does not mean _____.

Composition is not commutative:_____

Composition is used anytime a change in one quantity produces a change in another, which, in turn, produces a change in a third quantity. <u>Example</u>: The cost of travelling by car depends on the amount of gasoline used, and the amount of gasoline depends on the number of kilometers driven.

Problem #1

The function $f(d) = 0.73d$ converts Canadian dollars to Euros. The function $j(e) = 128e$ converts Euros to Japanese yen. Write a function that converts Can dollars to Japanese yen.

Problem #2
Consider $f(x) = \sqrt{x-2}$ and $g(x) = x^2$

Determine the following and find the domain, where necessary:

a) $(fog)(2)$

b) $(fog)(4)$

c) $(gof)(3)$

d) $(fog)(x)$

e) $(gof)(x)$

f) $(gog)(x)$

Problem #3

Given $f(x) = x + 1$ and $g(x) = \frac{1}{x-1}$, determine the following and find the domain:

a) $(fog)(x)$

b) $(gof)(x)$

c) $(gog)(x)$

When **decomposing** a composite function, we ask what function is on the inside and what function is on the outside. There may be multiple answers to the question.

Problem #4

Find two functions, f and g, both containing the variable x, to make the following true.

a) $(fog)(x) = 3x$

b) $(gof)(x) = (\sqrt{x} - 2)^3 - 1$

c) $(fog)(x) = \sqrt{x + 3}$

d) $(fog)(x) = x^2 - 2x + 2$

Problem #5

Sketch the graph of $(f \circ g)(x)$ if $g(x) = \sqrt{x + 3}$ and $f(x) = x^2 + 1$.