

8.2 – Properties of Chords in a Circle

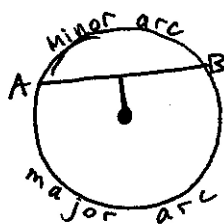
Focus: Relate a chord, its perpendicular bisector, and the centre of the circle, then solve problems.

Main Ideas:

Warmup:

Draw a circle freehand. Label the centre O. Draw a chord AB in your circle. Label the minor arc and the major arc.

- Draw a line from O to the halfway point of your chord.
- What kind of angle do you make?



(b) a 90° angle

What is a chord?

a line that joins two points on a circle

What is a diameter?

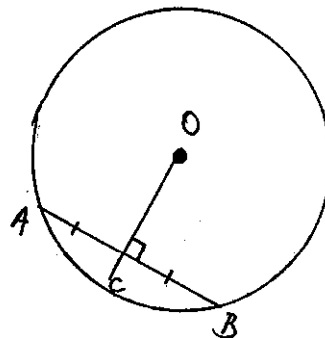
a chord that passes through the centre of the circle.

What is a perpendicular bisector?

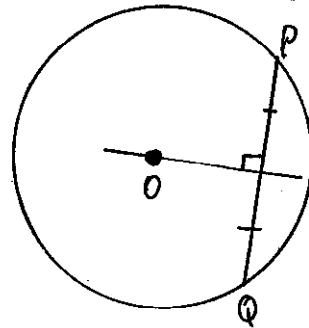
A line that intersects a chord at 90° and divides the chord into two equal parts.

What are the three Chord Properties?

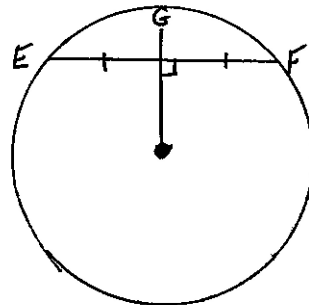
- The perpendicular from the centre of a circle to a chord bisects the chord; that is, the perpendicular divides the chord into two equal parts.



2. The perpendicular bisector of a chord in a circle passes through the centre of the circle.

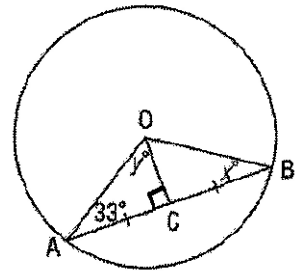
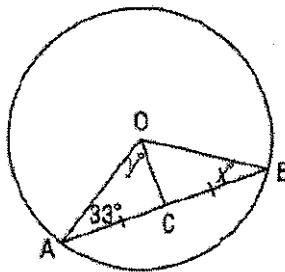


3. A line that joins the centre of a circle and the midpoint of a chord is perpendicular to the chord.



Ex1

Point O is the centre of a circle, and line segment OC bisects chord AB. $\angle OAC = 33^\circ$. Determine the values of x° and y° .

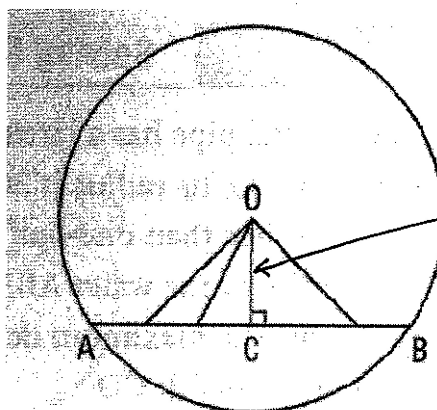


$\angle OCA = 90^\circ$ due to Perpendicular to Chord Properties so
 $y^\circ = 180 - 90 - 33 = \underline{57^\circ}$

$\triangle OAC$ same as $\triangle OBC$ so $x^\circ = \underline{33^\circ}$

An important point to consider:

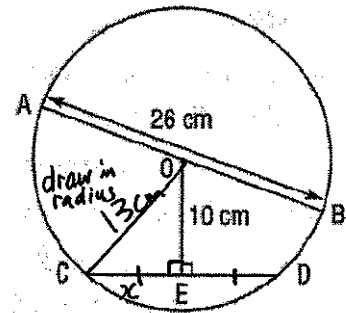
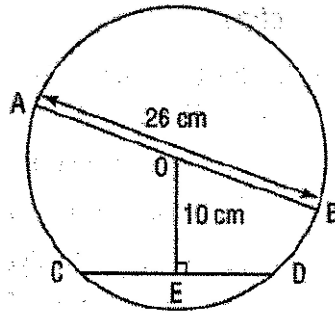
Many line segments can be drawn from O, the centre of a circle, to a chord AB. The *distance* from O to AB is defined as the shortest distance. This distance is the length of the perpendicular from O to AB; that is, the length of OC.



distance from O to AB

Ex2

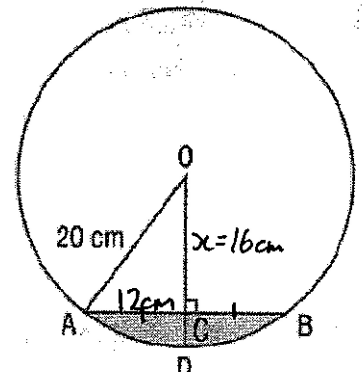
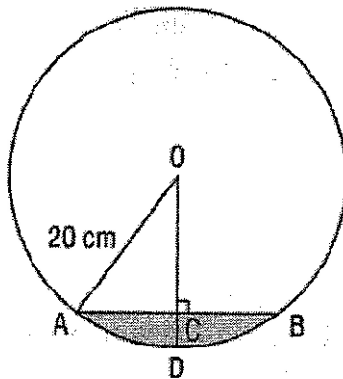
Point O is the centre of the circle. AB is a diameter with length 26cm. CD is a chord that is 10cm from the centre of the circle. What is the length of chord CD? Give the answer to the nearest tenth.



- if diameter = 26cm, radius = $26 \div 2 = 13$ cm
- OC is a radius, so $OC = 13$ cm
- $\angle OEC = 90^\circ$
- use Pythag to find length CE, call it x : $x^2 + 10^2 = 13^2$
 $x^2 + 100 = 169$
 $x^2 = 69$, so $x = \sqrt{69}$
 $x = 8.3$ cm
- CD is bisected by OE, so CD is exactly double CE,
 so $CD = 8.3 \times 2 = \underline{\underline{16.6}}$ cm

Ex3

A horizontal pipe has a circular cross-section, with centre O. Its radius is 20cm. Water fills less than one-half of the pipe. The surface of the water AB is 24cm wide. Determine the maximum depth of the water, which is the depth CD.



- if $AB = 24$ cm, AC is half at 12cm. Use pythag to find OC, call it x : $x^2 + 12^2 = 20^2$
- $x^2 + 144 = 400$
 $x^2 = 256$
 $x = \sqrt{256}$
 $x = 16$ cm
- OD is a radius = 20cm
 $OC = 16$ cm
 so $CD = 20 - 16 = 4$ cm
 The water depth is 4cm.

Reflection: What is the relationship between the centre of a circle, a chord, and the perpendicular bisector of the chord?