Polynomial Review

(3.1) 1. a) NOT – x^{-1} polyn. must have positive exp.
   b) 2
   c) 6
   d) NOT $x^{\frac{1}{3}}$ polyn. must have whole # exp.

2.

	LC	const
a)	–1	3
b)	9	5
c)	3	1
d)	–2	9

3. a) odd   NOT
   b) even   min
   c) odd.   NOT
   d) even   max

4.

	x int	D	R
a)	–4, –5, 2	$x \in \mathbb{R}$	$y \in \mathbb{R}$
b)	–6, 0	$x \in \mathbb{R}$	$y \leq 9$
c)	–1, –2, 5, 6	$x \in \mathbb{R}$	$y \geq -12.25$
d)	0, –3, –5, 7	$x \in \mathbb{R}$	$y \in \mathbb{R}$

5. a) $x \to 3$ possible $y = 3$
   b) $x \to 4$ " $y = 5$
   c) $x \to 4$ " $y = 1$
   d) $x \to 2$ " $y = 0$

6. a) quartic, even, both ends down, 4 poss. x int
      max value, y int = –5
   b) quintic, odd, – to +, 5 poss x int
      no max/min, y int = 12

7. a) odd degree, $LC = neg^+$ ↘_

b) pos x int 3

c) y int $= -2(1)(-2)(9) = 36$

d)
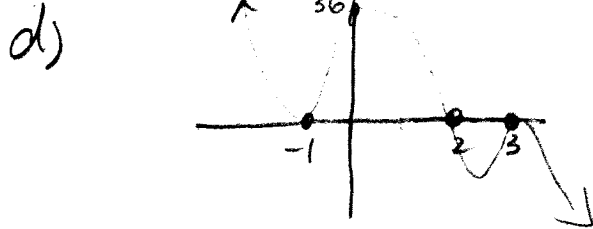
8. a) quartic b) quintic c) cubic d) quad

9. a) 2 b) $LC = -4.9$ const $= 60$ — y int.

c) $t \geq 0$ but must stop as object hit ground
time so $h = 0$
Quad I

$$0 = -4.9t^2 = 60$$
$$\frac{60}{4.9} = t^2$$
$$3.5 \sim t$$

then $0 \leq t \leq 3.5$

d) ends go down

10.

(3.2) 1.

$$\begin{array}{r} x + 3 \\ x-4 \overline{\smash{)}\ x^2 - x - 15} \\ \underline{-x^2 - 4} \\ 3x - 15 \\ \underline{-3x + 12} \\ -3 \end{array}$$

a) $\dfrac{x^2 - x - 15}{x-4} = (x+3) - \dfrac{3}{x-4}$

b) $x = 4$

c) $(x+3)(x-4) - 3 = x^2 - x - 15$

d) $x^2 - x - 12 - 3$
$x^2 - x - 15$ LS = RS ✓

2.
$$\begin{array}{r} x^3 - 6x^2 + \ldots - 5 \\ x+3\enclose{longdiv}{x^4 - 3x^3 + 2x^2 + \ldots 5x - 11} \\ \underline{x^4 + 3 \ldots} \\ -6x^3 - 18x^2 \\ \underline{-6x^3 - 18x^2} \\ 20x^2 + 55x \\ \underline{20x^2 + 60x} \\ -5x - 11 \\ \underline{-5x - 15} \\ 4 \end{array}$$

a) $\dfrac{x^4 - 3x^3 + \ldots}{x+3} = (x^3 - 6x^2 + \ldots - 5) + \dfrac{4}{x+3}$

b) $x \neq -3$

3. a)
$$\begin{array}{r} \boxed{3x - 1} \\ x-4\enclose{longdiv}{3x^2 - 13x - 2} \\ \underline{} \\ -x - 2 \\ \underline{-x + 4} \\ -6 \end{array}$$

b)
$$\begin{array}{r} 2x^2 - 20x + 85 \\ x+5\enclose{longdiv}{2x^3 - 10x^2 - 15x - 20} \\ \underline{2x^3 + 10x^2} \\ -20x^2 - 15x \\ \underline{-20x^2 - 100x} \\ 85x - 20 \\ \underline{85x + 425} \\ -445 \end{array}$$

c)
$$\begin{array}{r} 2w^3 - 3w^2 + 4w - 10 \\ w+3\enclose{longdiv}{2w^4 + 3w^3 - 5w^2 + \ldots} \\ \underline{2w^4 + 6w^3} \\ -3w^3 \\ \underline{-3w^3 - 9w^2} \\ 4w^2 + \ldots \\ \underline{4w^2 + 12w} \\ -10w - 27 \\ \underline{-10w + \ldots} \\ 3 \end{array}$$

4a)

$$3w^2 + 10w + 52$$

$$w - 5 \overline{)\, 3w^3 - 5w^2 + 2w - 27}$$

$$-\ 3w^3 + 15w^2$$

$$10w^2 + 2w$$

$$-\ 10w^2 + 50w$$

$$52w - 27$$

$$-\ 52w + 260$$

$$\boxed{233}$$

b)

$$3x^2 - 10x + 5$$

$$x + 1 \overline{)\, 2x^3 - 8x^2 - 5x - 2}$$

$$-\ 2x^3 + 10x$$

$$-10x^2 - 5x$$

$$-10x^2 - 10x$$

$$5x - 2$$

$$-\ 5x + 5$$

$$\boxed{-7}$$

c)

$$3x - 19$$

$$x + 2 \overline{)\, 3x^2 - 13x - 2}$$

$$-3x^2 + 6x$$

$$-19x - 2$$

$$-19x - 38$$

$$\boxed{36}$$

5. a)

$$-2 \,\big|\, \begin{array}{rrrrr} 4 & 3 & -7 & 2 & -1 \\ & -8 & 10 & -6 & 8 \\ \hline 4 & -5 & 3 & -4 & 7 \end{array}$$

$$(4w^3 - 5w^2 + 3w - 4 \; R\,7$$

b)

$$2 \,\big|\, \begin{array}{rrrrr} 1 & 2 & -8 & -5 & -2 \\ & 2 & 8 & 0 & -10 \\ \hline 1 & 4 & 0 & -5 & -12 \end{array}$$

$$x^3 + 4x^2 - 5 \; R\,-12$$

c)

$$-1 \,\big|\, \begin{array}{rrrrr} 5 & 0 & 2 & -1 & 4 \\ & -5 & 5 & -7 & 8 \\ \hline 5 & -5 & 7 & -8 & 12 \end{array}$$

$$5y^3 - 5y^2 + 7y - 8 \; R\,12$$

6 a)

$$5 \mid \begin{array}{ccc} 3 & -16 & 5 \\ & 15 & -5 \end{array}$$
$$\overline{\quad 3 \quad -1 \quad \boxed{0}\,R}$$

b)

$$-3 \mid \begin{array}{cccc} 2 & -3 & -5 & 6 \\ & -6 & 27 & -66 \end{array}$$
$$\overline{\quad 2 \quad -9 \quad 22 \quad -60 \quad \boxed{179}\,R}$$

c)

$$2 \mid \begin{array}{cccc} 4 & 5 & 0 & -7 \\ & 8 & 26 & 52 \end{array}$$
$$\overline{\quad 4 \quad 13 \quad 26 \quad \boxed{45}\,R}$$

7 a) $x = -2$ $-4(-2)^4 - 3(-2)^3 + 2(-2)^2 \cdots = -25$

b) $7(-2)^5 + 5(-2)^4 + \cdots - 3 = -44$

c) $8(-2)^3 - 1 = -65$

8 a)

$$-1 \mid \begin{array}{cccc} 3 & -4 & 6 & -9 \\ & -3 & 7 & -13 \end{array}$$
$$\overline{\quad 3 \quad -7 \quad 13 \quad \boxed{-22}}$$

b)

$$2 \mid \begin{array}{ccc} 3 & -8 & 4 \\ & 6 & -4 \end{array}$$
$$\overline{\quad 3 \quad -2 \quad \boxed{0}}$$

c)

$$-5 \mid \begin{array}{cccc} 6 & -5 & -7 & 9 \\ & -30 & 175 & -840 \end{array}$$
$$\overline{\quad 6 \quad -35 \quad 168 \quad \boxed{-831}}$$

9 $\quad 2(-3)^3 + 5(-3)^2 - k(-3) + 9 = 6$
$$-54 + 45 + 3k + 9 = 6$$
$$3k = 6$$
$$k = 2$$

10. $\quad 4k^2 - 8k - 20 = 12$
$$4k^2 - 8k - 32 = 0$$
$$4(k^2 - 2k - 8) = 0$$
$$4(k-4)(k+2) = 0$$
$$k = 4, -2$$

③·③

1. a) $x = 6$ b) $x = -7$ c) $x = 2$ d) $x = -5$
    $x - 6$ $x + 7$ $x - 2$ $x + 5$

2. a) $-4 - 3 + 2 - 1 + 5 = -1$ X
  b) $7 + 5 + 23 + 8 = 43$ X
  c) $2 - 3 - 5 + 6 - 1 = -1$ X
  d) $2 + 5 - 7 = 0$ ✓

3. a) $-3(-8) + 2(4) - 20 + 5 = 24 + 8 - 20 + 5 = 17$ X
  b) $5(4) + 6(-2) - 8 = 20 - 12 - 8 = 0$ ✓
  c) $2(16) - 3(-8) - 5(4) = 32 + 24 - 20 = 36$ X
  d) $3(-8) + 24 - 2 = -24 + 24 - 2 = -2$ X

4. a) $12 \rightarrow \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
  b) $-6 \rightarrow \pm 1, \pm 2, \pm 3, \pm 6$
  c) $-25 \rightarrow \pm 1, \pm 5, \pm 25$
  d) $10 \rightarrow \pm 1, \pm 2, \pm 5, \pm 10$

5. $x + 3$ $x - 4$ $x + 1$
  zeros $\rightarrow -3, \quad 4, \quad -1$

6. a)
```
1 | 1   2   -13   10
  |     1    3   -10
  ----------------------
    1   3   -10    0
```
$x^2 + 3x - 10$
$(x - 1)(x + 5)(x - 2)$

b)
```
3 | 1   -7    3    63   -108
  |       3  -12   -27    108
  ------------------------------
    1   -4   -9    36     0
```
$x^3 - 4x^2 - 9x + 36$
```
4 | 1   -4   -9    36
  |      4    0   -36
  --------------------
    1    0   -9     0
```
$x^2 - 9$
$(x - 3)(x - 4)(x - 3)(x + 3)$

c)
```
-1 | 1   -1   -26   -24
   |      -1    2    24
   ---------------------
     1   -2   -24    0
```
$x^2 - 2x - 24$
$(x + 1)(x - 6)(x + 4)$

6d)

$$1 \;|\; \begin{array}{ccccc} 1 & 0 & -26 & 0 & 25 \\ & 1 & 1 & -25 & -25 \\ \hline 1 & 1 & -25 & -25 & 0 \end{array}$$

$x^3 + x^2 - 25x - 25$

$$-1 \;|\; \begin{array}{cccc} 1 & 1 & -25 & -25 \\ & -1 & 0 & 25 \\ \hline 1 & 0 & -25 & 0 \end{array}$$

$x^2 - 25$

$(x-1)(x+1)(x-5)(x+5)$

7.a)

$$-1 \;|\; \begin{array}{cccc} 1 & 1 & -16 & -16 \\ & & & 0 \\ \hline 1 & 0 & -16 & 0 \end{array}$$

$x^2 - 16$

$(x+1)(x-4)(x+4)$

b)

$$4 \;|\; \begin{array}{cccc} 1 & -2 & -6 & 8 \\ & 4 & 8 & 8 \\ \hline 1 & 2 & 2 & 0 \end{array}$$

$(x-4)(x^2 + 2x + 2)$

c)

$$3 \;|\; \begin{array}{cccc} 1 & 6 & -7 & -60 \\ & 3 & 27 & 60 \\ \hline 1 & 9 & 20 & 0 \end{array}$$

$x^2 + 9x + 20$

$(x-3)(x+4)(x+5)$

d)

$$5 \;|\; \begin{array}{cccc} 1 & 0 & -27 & 10 \\ & 5 & 25 & -10 \\ \hline 1 & 5 & -2 & 0 \end{array}$$

$(x-5)(x^2 + 5x - 2)$

8. a)

$$-1 \;|\; \begin{array}{ccccc} 1 & 4 & -7 & -34 & -24 \\ & -1 & -3 & 10 & 24 \\ \hline 1 & 3 & -10 & -24 & 0 \end{array}$$

$x^3 + 3x^2 - 10x - 24$

$$3 \;|\; \begin{array}{cccc} 1 & 3 & -10 & -24 \\ & 3 & 18 & 24 \\ \hline 1 & 6 & 8 & 0 \end{array}$$

$x^2 + 6x + 8$

$(x+1)(x-3)(x+4)(x+2)$

b)

$$2 \;|\; \begin{array}{cccccc} 1 & 3 & -5 & 15 & 4 & 12 \\ & 2 & 10 & 10 & -10 & -12 \\ \hline 1 & 5 & 5 & -5 & -6 & 0 \end{array}$$

$x^4 + 5x^3 + 5x^2 - 5x - 6$

$$-1 \;|\; \begin{array}{ccccc} 1 & 5 & 5 & -5 & -6 \\ & -1 & -4 & -1 & 6 \\ \hline 1 & 4 & 1 & -6 & 0 \end{array}$$

$x^3 + 4x^2 + x - 6$

$$1 \;|\; \begin{array}{cccc} 1 & 4 & 1 & -6 \\ & 1 & 5 & 6 \\ \hline 1 & 5 & 6 & 0 \end{array}$$

(x-2)
(x+1)
(x-1)
$(x+3)(x+2) \leftarrow x^2 + 5x + 6$

9. a)

$$-2 \,|\, \underline{\begin{array}{ccc} 1 & -8 & -20 \\ & -2 & 20 \end{array}}$$
$$\qquad\quad 1 \quad -10 \quad ⓪$$

$$x^2 - 10x$$

$$(\widehat{x+2}) \times (x-\widehat{10})$$

b)

$$7 \,|\, \underline{\begin{array}{ccc} 1 & -3 & -k \\ & 7 & 28 \end{array}}$$
$$\qquad\quad \quad 4$$

$$-k + 28 = 0$$
$$k = 28$$

10. a) $k(3)^3 - 10(3)^2 + 2(3) + 3 = 0$

$$27k - 90 + 6 + 3 = 0$$
$$27k = 81$$
$$k = \frac{81}{27} \quad ③$$

b) $4(3)^4 - 3(3)^3 - 2(3)^2 + k(3) - 9 = 0$

$$324 - 81 - 18 + 3k - 9 = 0$$
$$3k = -216$$
$$k = -72$$

③.④

1. a) $x = -5, -2, 3, 6$

   b) $x^3 = 27$ c) $x = -\frac{1}{3}, 4, 7$ d) $x = 0, -4, -2$
      $x = 3$

2. a) $-4, -3, 2$ b) on $(-4, -3)$ $(2, \infty)$
                      $-4 < x < -3$, $x > 2$
                   c) $x < -4$, $-3 < x < 2$
                      $(\infty, -4)$ ' $(-3, 2)$

3. a) 3 b) — $\underset{\text{max}\atop\text{slope}}{\swarrow}$ c) $x = -4, -3, -2$ factor $(x+4)(x+3)(x+2)$

   d) $+ \rightarrow$ $x < -4$ $(\infty, -4)$ $- \rightarrow$ $-4 < x < -3$ $(-4, -3)$
                   $-3 < x < -2$ $(-3, -2)$ $x > -2$ $(-2, \infty)$

4.

$x/4$	$x/4, -2y$	$x/4 - 1, -2y - 5$
$(=\frac{1}{2}, -8)$	$(-\frac{1}{2}, 16)$	$(-1\frac{1}{2}, 11)$
$(-\frac{1}{4}, -1)$	$(-\frac{1}{4}, 2)$	$(-1\frac{1}{4}, -3)$
$(0, 0)$	$(0, 0)$	$(-1, -5)$
$(\frac{1}{4}, 1)$	$(\frac{1}{4}, -2)$	$(-3/4, -7)$
$(\frac{1}{2}, 8)$	$(\frac{1}{2}, -16)$	$(-1/2, -21)$

5

$2x$	$2x, \frac{1}{4}y$	$2x+9, \frac{1}{4}y+3$
$(-4, -16)$	$(-4, -4)$	$(5, -1)$
$(-2, 1)$	$(-2, \frac{1}{4})$	$(7, 3\frac{1}{4})$
$(0, 0)$	$(0, 0)$	$(9, 3)$
$(2, 1)$	$(2, \frac{1}{4})$	$(11, 3\frac{1}{4})$
$(4, 16)$	$(4, 4)$	$(13, 7)$

6. a) 4   b) + opening up   c) $x = -6^{②}, -3, 1$ factors $(x+6)^2(x+3)(x-1)$

d) $+ \rightarrow$ $x < -6 \rightarrow (-\infty, -6)$

$-6 < x < -3 \rightarrow (-6, -3)$

$x > 1 \rightarrow (1, \infty)$

$- \rightarrow$ $-3 < x < 1 \rightarrow (-3, 1)$

7a)
$$1 \mid \begin{array}{cccc} 1 & 4 & -1 & -4 \\ & 1 & 5 & 4 \\ \hline 1 & 5 & 4 & 0 \end{array}$$

$x^2 + 5x + 4 = (x+4)(x+1)$

$x = 1, -4, -1$

b) degree = 3
end $-$ to $+$

c) -4

d)
$$\underline{\quad - \quad -4 \quad + \quad -1 \quad - \quad 1 \quad + \quad}$$

$+ \rightarrow$ $-4 < x < -1 \rightarrow (-4, -1)$

$x > 1 \rightarrow (1, \infty)$

$- \rightarrow$ $x < -4 \rightarrow (-\infty, -4)$

$-1 < x < 1 \rightarrow (-1, 1)$

8 —

9a) $y = a(x-3)^2(x+1)$

$18 = a(0-3)^2(0+1)$

$2 = a$

$y = 2(x-3)^2(x+1)$

b) $y = a(x+2)^3(x-4)^2$

$-32 = a(2)^3(-4)^2$

$-32 = a(128)$

$-\frac{1}{4} = a$

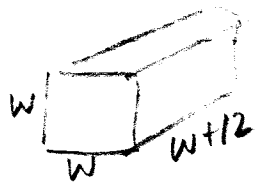$y = -\frac{1}{4}(x+2)^3(x-4)^2$

c) $y = a(x+1)^2(x-5)^2$

$-10 = a(1)(25)$

$-\frac{2}{5} = a$

$y = -\frac{2}{5}(x+1)^2(x-5)^2$

10. $x(x+1)(x+2) = -504$

$(x^2+x)(x+2)$

$x^3 + 2x^2 + x^2 + 2x$

$x^3 + 3x^2 + 2x + 504 = 0$

$$\begin{array}{c|cccc} -9 & 1 & 3 & 2 & 504 \\ & & -9 & 54 & -504 \\ \hline & 1 & -6 & 56 & 0 \end{array}$$

$\Rightarrow x = -9, -8, -7$
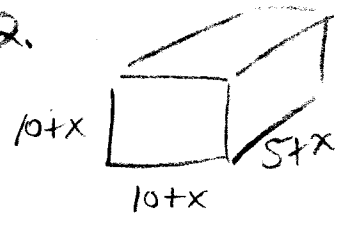
11.

$w \quad h \quad \ell$

$w(w)(w+12) = 135$

$w^3 + 12w^2 - 135 = 0$

$$\begin{array}{c|cccc} 3 & 1 & 12 & 0 & -135 \\ & & 3 & 45 & 135 \\ \hline & 1 & 15 & 45 & 0 \end{array}$$

$w^2 + 15w + 45$

$W = 3$
$h = 3$
$\ell = 15$

12.

$(10+x)(10+x)(5+x) = 1008$

$(100 + 20x + x^2)(5+x) - 1008 = 0$

$500 + 100x + 5x^2 + 100x + 10x^2 + x^3 - 1008 = 0$

$x^3 + 25x^2 + 200x - 508 = 0$

$$\begin{array}{c|cccc} 2 & 1 & 25 & 200 & -508 \\ & & 2 & 54 & 508 \\ \hline & 1 & 27 & 254 & 0 \end{array}$$

$10+x = 12$
$10+x = 12$
$5+x = 7$

Radical Functions Review

1. —

2. a) stretch vert 3, shift right 5 $D \to x > 5$ $R \to y$
   b) reflect in x axis   shift up 7
   c) compress vert 0.25, hor $\frac{1}{0.25} = 4$, shift down 3
   d) reflect in y axis , shift left 1 , down 5

3. a) $y = 2\sqrt{x} - 1$ down 1
      D

   c) $y = 2\sqrt{x-1}$ right 1
      C   } no reflections (C OR D)

   b) $y = -2\sqrt{x} - 1$ reflect vert.
      A

   c) $y = 2\sqrt{-(x-1)}$ reflect hor.
      B

4. a) $y = 3\sqrt{\frac{1}{2}x}$

   c) $y = -\sqrt{3x} - 7$

   b) $y = \sqrt{-(x+2)} + 3$

   d) $y = 5\sqrt{4(x-6)}$ $\frac{1}{0.25}$

5. a) $y = 5\sqrt{x+7} - 2$ vert st 5   shift left 7   down 2

   b) $y = -4\sqrt{-x} + 8$ reflect vert.   stretch vert   reflect hor.   up 8

   c) $y = \sqrt{0.25(x-1)}$ hor. $\frac{1}{0.25} = 4$ st.   right 1

   d) $y = \sqrt{\frac{1}{3}(x+4)} - 3$ hor $\frac{1}{\frac{1}{2}} = 3$ st.   left 4   down 3

6. — a) $(x, y) \to (x, -y) \to (x+3, -y+5)$
      b) $(x, y) \to (x, 4y) \to (3x, 4y)$
      c) $(x, y) \to (x, -y) \to (\frac{x}{2}, -y)$

7 a) $(x, y) \rightarrow (x+4, 2y-5)$

b) $(x, y) \rightarrow (x, -3y+6)$

c) $(x, y) \rightarrow (2x, -y+1)$

d) $(x, y) \rightarrow (\frac{x}{2} - 3, y+9)$

8. a) $-x \geq 0$
$x \leq 0$
$y \geq -4$

b) $x - 4 \geq 0$
$x \geq 4$
$y \geq 0$

c) $x - 4 \geq 0$
$x \geq 4$
$y \leq 4$

d) $x \geq 0$
$y \leq 0$

9 a) $y = a\sqrt{b(x-7)} + 3$

pt $(8, 5)$ $5 = a\sqrt{b(8-7)} + 3$

$2 = a\sqrt{b}$

$y = 2\sqrt{x-7} + 3$

b) $y = a\sqrt{b(x-3)}$

$2 = a\sqrt{-b(-1)}$

$2 = a\sqrt{b}$ pt $(2, 2)$

$y = 2\sqrt{-(x-3)}$

c) $y = a\sqrt{b(x+5)}$

pt $(-3, 1)$ $1 = a\sqrt{b(2)}$

$1 = a\sqrt{2b}$

$y = \sqrt{\frac{1}{2}(x+5)}$

10 a) $y = \sqrt{-x-7}$

$y = \sqrt{-(x+7)}$

↑ hor reflect   ↑ left 7

b) $y = \sqrt{2x-6} + 5$

$y = \sqrt{2(x-3)} + 5$

hor comp $\frac{1}{2}$ right 3   up 5

c) $y = \sqrt{-x+5} + 7$

$y = \sqrt{-(x-5)} + 7$

hor reflect   right 5   up 7

(2.2) 1.

x	$f(x)$	$\sqrt{f(x)}$
-2	16	4
-1	8	2.8
0	4	2
1	1.96	1.4
2	1	1

2. a) $\sqrt{14}$ ✓ 3.74 → $(9, 3.74)$

b) $\sqrt{r}$ ✓ → $(p, \sqrt{r})$

c) $\sqrt{7}$ ✓ 2.65 → $(-2, 2.65)$

d) $\sqrt{-1}$ ✗

3 →

4. a) 　　b) $x \in \mathbb{R}$　c) →　d) $x \geq -4$
　　　　　　　　　　$y \in \mathbb{R}$　　　　　　　　$y \geq 0$

5. a) $x \geq 4$　　　b) $x \geq -9$　　　c) $x \geq 9$
　　$y \geq 0$　　　　　$y \geq 0$　　　　　$y \geq 0$
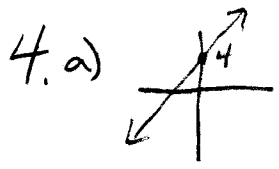
6. a) $x \in \mathbb{R}$　$x \geq -5$　b) $x \in \mathbb{R}$　$x \geq 3$　c) $x \in \mathbb{R}$　$x \leq -10$
　　$y \in \mathbb{R}$　$y \geq 0$　　　$y \in \mathbb{R}$　$y \geq 0$　　　$y \in \mathbb{R}$　$y \geq 0$
　　restriction on $\sqrt{}$

7. a) $x \leq -4$　　　b) $x \in \mathbb{R}$　　　c) $x \in \mathbb{R}$
　　$x \geq 4$　　　　　$y \geq 2.2$　　　　　$y \geq 4.2$
　　$y \geq 0$　　　　　　$(\sqrt{5})$　　　　　　$(\sqrt{18})$

8 —　　　　　　　9 —

10 invariant - never changing

$\boxed{2.3}$ 1. a) $\sqrt{x+1} + 3 = 5$　　b) $\sqrt{4-3x} = 2$
　　　　　$\sqrt{x+1} = 2$　　　　　　　$4-3x = 4$
　　　　　$x+1 = 4$　　　　　　　　　$-3x = 0$
　　　　　$x = 3$ ✓　　　　　　　　　$x = 0$ ✓

　c) $\sqrt{0.5(3x-2)} = -1$　　d) $-3\sqrt{x+2} + 4 = 1$
　　　$0.5(3x-2) = 1$　　　　　　　$-3\sqrt{x+2} = -3$
　　　　$3x-2 = 2$　　　　　　　　　$\sqrt{x+2} = 1$
　　　　　$3x = 4$　　　　　　　　　$x+2 = 1$
　　　　　　$x = \frac{4}{3}$ ✗　　　　　　　$x = -1$ ✓
　　　　NO SOLUTION

2. a) $y = \sqrt{5x^2 + 1} - x - 5$　　b) $y = x - 3 - \sqrt{2x-7}$
　 c) $y = \sqrt{13-4x^2} - 2 + x$　　d) $y = x + \sqrt{-2x^2 + 9} - 3$

3. a) $x = -4.4$　　b) $x = -9$　　c) $x = -7.2$　　d) $x = -3$

4a) $\sqrt{2x+1} = 3$

$2x+1 = 9$

$2x = 8$

$x = 4$ ✓

b) $\sqrt{x-3} = -4$

$x-3 = 16$

$x = 19$ X

c) $\sqrt{4(x+3)} = 6$

$4(x+3) = 36$

$x+3 = 9$

$x = 6$ ✓

d) $2\sqrt{x-1} = 10$

$\sqrt{x-1} = 5$

$x-1 = 25$

$x = 26$ ✓

5a) $x = \sqrt{x+2}$

$x^2 = x+2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$ X

b) $\sqrt{x+4} = x-8$

$x+4 = x^2 - 16x + 64$

$0 = x^2 - 17x + 60$

$(x-12)(x-5)$

$x = 12, 5$ X

c) $\sqrt{x-1} = x-3$

$x-1 = x^2 - 6x + 9$

$0 = x^2 - 7x + 10$

$(x-5)(x-2)$

$x = 5, 2$ X

d) $x-2 = \sqrt{x+10}$

$x^2 - 4x + 4 = x+10$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1)$

$x = 6, -1$ X

6.a) $\sqrt{x-2} = x-3$

$x-2 = x^2 - 6x + 9$

$= x^2 - 7x + 11$

$x = \dfrac{7 \pm \sqrt{49-44}}{2}$

$x = \dfrac{7 \pm \sqrt{5}}{2}$ $x = 4.6, 2.4$ X

b) $\sqrt{x+1} = 2x-5$

$x+1 = 4x^2 - 20x + 25$

$= 4x^2 - 21x + 24$

$x = \dfrac{21 \pm \sqrt{441-384}}{8}$

$x = \dfrac{21 \pm \sqrt{57}}{8}$ $x = 3.6, 1.6$ X

c) $x\sqrt{3} = x-4$

$3x^2 = x^2 - 8x + 16$

$2x^2 + 8x - 16 = 0$

$2(x^2 + 4x - 8) = 0$

$x = \dfrac{-4 \pm \sqrt{16+32}}{2}$

$x = \dfrac{-4 \pm \sqrt{48}}{2}$ $x = 1.5, -1.5$

6d) $\sqrt{x^2-4} = 2x-10$

$x^2-4 = 4x^2-40x+100$

$= 3x^2-40x+104$

$x = \dfrac{40 \pm \sqrt{1600-1248}}{6}$

$x = \dfrac{40 \pm \sqrt{352}}{6}$ $x = 9.8, 3.5$

7. a) $\sqrt{1+y} = \dots$

   $x = -3x$

b) $\sqrt{1-y} = -2$

   $\sqrt{} = -$ NEVER!
   in $\mathbb{R}$

8. $10 = 3\sqrt{d}$

   $\dfrac{10}{3} = \sqrt{d}$

   $\dfrac{100}{9} = d$

   $11m = d$

9. a) $r = \dfrac{1}{2}\sqrt{\dfrac{172}{\pi}}$

   $r = 3.7cm$

b) $3.3 = \dfrac{1}{2}\sqrt{\dfrac{A}{\pi}}$

   $6.6 = \sqrt{\dfrac{A}{\pi}}$

   $43.56 = \dfrac{A}{\pi}$

   $137cm^2 = A$

10. $\sqrt{x+\sqrt{x-2}}$

   $x + \sqrt{x-2} = 4$

   $\sqrt{x-2} = 4-x$

   $x-2 = 16-8x+x^2$

   $= x^2-9x+18$

   $(x-6)(x-3)$

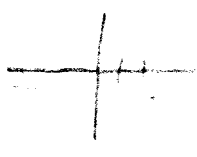 $x = 6, 3$

Rational Functions Review

9.1 1. a) $y = \frac{4}{x} + 3$ $\uparrow$ up3 B b) $y = \frac{4}{x+3}$ $\leftarrow$ l3 C c) $\frac{4}{x-3}$ R3 D d) $\frac{4}{x} - 3$ D3 **A**

2. $y = \frac{5}{x-2} + y = 0$ $\quad x = 2$ $\quad$ NPV — non permissable values

$D - x \neq 2$
$R - y \neq 0$

vert asy — $x = 2$
hor asy — $y = 0$

3. a) D $x \neq 1$ b) D $x \neq 0$ c) D $x \neq -4$ d) D $x \neq -2$
   R $y \neq 0$ R $y \neq 6$ R $y \neq -2$ R $y \neq 3$

y int = -3 y int = NO   y int $= -\frac{3}{4}$ y int $= 8.5$
x int = NO   x int $= -\frac{1}{3}$ x int $= -1.5$ x int $= -\frac{17}{8} = -2.125$

asy $\begin{cases} x=1 \\ y=0 \end{cases}$ asy $\begin{cases} x=0 \\ y=6 \end{cases}$ asy $\begin{cases} x=-4 \\ y=-2 \end{cases}$ asy $\begin{cases} x=-2 \\ y=3 \end{cases}$

4. a) $x = 1$ y int $= -5$ b) $x = -2$ y int $= \frac{2}{3}$
      $y = 2$ x int $> -\frac{5}{2}$ $y = 4$ x int $= \frac{3}{4}$

5. a) $y = \frac{a}{x} \rightarrow y = \frac{3}{x}$ b) $y = \frac{a}{x} \rightarrow y = \frac{4}{x}$

pt $\rightarrow$ $3 = \frac{a}{1}$ pt $\rightarrow$ $2 = \frac{a}{2}$
(1,3)   (2,2)
    $a = 3$ $4 = a$

c) $y = \frac{a}{x-5} \rightarrow y = \frac{2}{x-5}$ d) $y = \frac{a}{x+4} \rightarrow y = \frac{-2}{x+4}$

pt $2 = \frac{a}{6-5}$ pt $2 = \frac{a}{-5+4}$
(6,2)   (-5,2)
    $2 = a$ $-2 = a$

6. a) $7 = \dfrac{a}{6-5} + k$ $\bigg\}$ $1 = \dfrac{a}{4-5} + k$ b) →

① $7 = a + k$ $\bigg\}$ ② $1 = -a + k$

①+② $\quad 7 = a + k$

$\quad\quad\quad \underline{1 = a + k}$

$\quad\quad\quad \dfrac{8 = 2k}{\boxed{4 = k}}$

→ ① $7 = a + 4$

$\boxed{3 = a}$

7. →

8. →

9. a) $d = st \rightarrow t = \dfrac{d}{s}$

b) $t = \dfrac{351}{65} = 5.4\,h$

c) $s = \dfrac{351}{s} \longrightarrow s = \dfrac{351}{5} = 70.2\,km/h$

(9.2) 1. →

2. $y = \dfrac{x+2}{(x+2)(x+1)} \rightarrow y = \dfrac{1}{x+1} \rightarrow$ asy. at $x = -1$

$\rightarrow$ pt of disc. also $x = -2$ $\quad y = \dfrac{1}{-2+1} \Rightarrow \boxed{-1 = y}$

3. →

4. →

5. → a) $y = \dfrac{x(x+5)}{(x+5)(x+2)} = \dfrac{x}{x+2}$ $x \neq -5$ $x \neq -2$

pt disc. $x = -5$ ↓ asy $x = -2$

$y = \dfrac{-5}{-5+2} = \dfrac{5}{3}$

xint $y = 0$

$0 = \dfrac{x(x+5)}{(x+5)(x+2)}$

$0 = x(x+5)$

$\boxed{x = 0}$ ✗

yint $x = 0$

$y = \dfrac{0+0}{0+0+10} = 0$

5b) $y = \dfrac{(x-4)(x-3)}{(x-3)(x+3)} \rightarrow y = \dfrac{x-4}{x+3}$

pt disc.   asy

$x = 3$ $x \neq -3$

$y = \dfrac{3-4}{3+3} = \dfrac{-1}{6}$

xint $y = 0$

$0 = (x-4)(x-3)$

$x = 4, \cancel{3}$

yint $x = 0$

$y = \dfrac{0-0+12}{0-9} = \dfrac{12}{-9} \left(\dfrac{4}{-3}\right)$

c) $y = \dfrac{(x+4)(x+1)}{x+1} \rightarrow y = x+4$

pt disc $x = -1$

$y = -1+4 = 3$

xint $y = 0$

$y = (x+4)(x+1)$

$x = -4, \cancel{X}$

yint $x = 0$

$y = \dfrac{0+0+4}{0+1} = 4$

d) $y = \dfrac{(2x-1)(x+3)}{x+3} \rightarrow y = 2x-1$

pt disc. $x = -3$

$y = 2(-3)-1 = -7$

xint $y = 0$

$0 = (2x-1)(x+3)$

$x = \frac{1}{2}$ $\cancel{x=-3}$

yint $x = 0$

$y = \dfrac{0+0-3}{0+3} = -1$

6. →

7. a) $y = \dfrac{x+4}{(x-4)(x+1)}$ asym.   b) $\dfrac{x+4}{(x+4)(x+1)}$ pt + asym.

c) $\dfrac{x(x+4)}{x+4}$ pt.

8a) line → $y = x+2$ pt disc $x = 3 \rightarrow (x-3)$ $\frac{num}{+denom}$

b) line → $y = x-2$ pt disc $x = -2 \rightarrow (x+2)$ $\frac{num}{+denom}$

c) asym $x = 4 \rightarrow x - 4$ denom   pt disc. $x = -4 \rightarrow x+4$ $\frac{num}{+denom}$

d) asy $x = -3 \rightarrow x+3$ denom

pt disc. $x = -5 \rightarrow x+5$ $\frac{num}{+denom}$

9. asym $x=2 \rightarrow x-2$ denom

pt disc $x=-2.5 \rightarrow x+2.5$ num + denom.

another pt $(6, -3)$

$$y = \frac{a(x+2.5)}{(x-2)(x+2.5)}$$

$$-3 = \frac{a(6+2.5)}{(6-2)(6+2.5)}$$

$$-12 = a$$

$x+2.5$ can be.

$x + \dfrac{5}{2}$

$2x+5$

(9.3) 1.a)

$$\frac{2}{x-1} - 5 = \frac{4}{x-1}$$

mult all by $(x-1)$ LCD to get rid of denom.

$\boxed{X \neq 1}$

$$2 - 5(x-1) = 4$$

$$2 - 5x + 5 = 4$$

$$-5x = -3$$

$$\boxed{x = 3/5}$$

b)

$$\frac{3}{x+5} + \frac{1}{2} = \frac{x+3}{x+5}$$

mult all by $2(x+5)$ LCD

$\boxed{X \neq -5}$

$$2(3) + 1(x+5) = (x+3)2$$

$$6 + x+5 = 2x+6$$

$$\boxed{5 = x}$$

c)

$$\frac{8}{x} + \frac{x+6}{3x} + \frac{x-4}{6x} = \frac{8}{9}$$

$LCD = 18x$

$\boxed{X \neq 0}$

$$18(8) + 6(x+6) + 3(x-4) = 2x(8)$$

$$144 + 6x + 36 + 3x - 12 = 16x$$

$$168 = 7x$$

$$\boxed{24 = X}$$

d)

$$\frac{x^2+2}{x} = \frac{2x+1}{2}$$

cross mult
$x \neq 0$

$$2x^2 + 4 = 2x^2 + x$$

$$\boxed{4 = x}$$

2a) $x = \dfrac{13}{x-9} - 3$ $x \neq 9$

$x^2 - 9x = 13 - 3x + 27$

$x^2 - 6x - 40 = 0$

$(x-10)(x+4) = 0$

$\boxed{x = 10, -4}$

b)

$x(x-3) = x + 5 + 4(x-3)$ $x \neq 3$

$x^2 - 3x = x + 5 + 4x - 12$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$\boxed{x = 7, 1}$

c) $(x+4)(x-7) = 4x + 2$ $x \neq 7$

$x^2 - 3x - 28 = 4x + 2$

$x^2 - 7x - 30 = 0$

$(x-10)(x+3) = 0$

$\boxed{x = 10, -3}$

d) $(x+3)(2-x) = x^2$ $x \neq 2$

$-x^2 - x + 6 = x^2$

$0 = 2x^2 + x - 6$

$= (2x - 3)(x + 2)$

$\boxed{x = 3/2, -2}$

3a) $\dfrac{5}{x} + x - 6 = 0$ mult all by x

$x \neq 0$

$5 + x^2 - 6x = 0$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1)$ $x = 5, 1$

b) →

c) xint are the roots also known as ZEROS/SOLUTION

4. a) $y_1 = 3x$
$y_2 = \dfrac{6x}{2x-5}$ } find pts of intersection

5. a) $\dfrac{x}{(x-3)} + 4 - x$

b) $\dfrac{4}{(x+1)} - \dfrac{2}{(x-1)}$

6. a) $x - 1 = \dfrac{x}{x-4}$

$x^2 - 5x + 4 = x$

$x^2 - 6x + 4 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 16}}{2}$

$x = \dfrac{6 \pm \sqrt{20}}{2}$ < $\begin{array}{l} 5.24 \\ 0.76 \end{array}$

b) $x + 3 = \dfrac{x+2}{x-1}$

$x^2 + 2x - 3 = x + 2$

$x^2 + x - 5 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 20}}{2}$

$x = \dfrac{1 \pm \sqrt{21}}{2}$ < $\begin{array}{l} 1.79 \\ -2.79 \end{array}$

c) $\dfrac{3}{5x} + x = 5$

$3 + 5x^2 - 2x = 25x - 10$

$5x^2 - 27x + 13 = 0$

$x = \dfrac{27 \pm \sqrt{729 - 260}}{10}$

$x = \dfrac{27 \pm \sqrt{469}}{10}$ < $\begin{array}{l} 4.87 \\ 0.53 \end{array}$

7. a) $y_1 = 2x \div (x-1) + 3x - (x-3) \div (x+1)$ $\longrightarrow$
b) $y_1 = 4 - 3 \div (x-1) - 9 + (x+3) \div x$ $\longrightarrow$

8. $\dfrac{18}{(n-3)(n+3)} + 1 = \dfrac{11}{n+3}$ $LCD = (n+3)(n-3)$ X all by

$18 + n^2 - 9 = n(n-3)$

$n^2 + 9 = n^2 - 3n$

$9 = -3n$

$-3 = n$ but $\nearrow$ $n \neq 3, -3$

$\downarrow$

So NO SOLUTION

9. Carmen c hrs
James $c + 9$ hrs
LCD $c(c+9)$

$$\frac{20}{c} + \frac{20}{c+9} = 1 \text{ job}$$

$$20c + 180 + 20c = c^2 + 9c$$

$$0 = c^2 - 31c - 180$$

$$c = \frac{31 \pm \sqrt{961 + 720}}{2}$$

$$c = 36, \cancel{-5}$$

Carmen - 36 hrs, James 45 hrs

(1.1) Transformations Review

1 first up 3   2nd down 2

2 " left 4   " right 5

3 a) $(x, y) \rightarrow (x+3, y+6)$
  b) $(x, y) \rightarrow (x, y-4)$
  c) $(x, y) \rightarrow (x-2, y+4)$
  d) $(x, y) \rightarrow (x+1, y-2)$

4 a) left 2 up 3   b) right 5 down 7
  c) left 4   d) up 6

5 $(x, y) \rightarrow (x-3, y+5)$
  A $(-2, 0) \rightarrow (-5, 5)$
  B $(-1, 3) \rightarrow (-4, 8)$
  C $(2, 3) \rightarrow (-1, 8)$
  D $(5, 1) \rightarrow (2, 6)$

6. a) $A (2, 0) \rightarrow A' (1, -4)$ (b) $y = f(x+1) - 4$
     $(x-1, y-4)$

7 a) $h = -3, k = 2$ $y = (x+3)^2 + 2$ b) $h = 5, k = -1$ $y = |x-5| - 1$

  c) $h = 9, k = -5$ $y = g(x-9) - 5$ d) $h = 4, k = 9$ $y = \frac{1}{x+4} + 9$

8 $y=x^2$
$x=3 \rightarrow y = 9 \boxed{-4} = 5$

(1.2) 1 → invariant — shared/same

2 →

3 → $(x,y) \rightarrow (x, 2y)$ $A(-2,0)\ B(-1,6)\ C(2,6)\ D(5,2)$

4. $(x,y) \rightarrow (3x, y)$ $A(-6,0)\ B(-3,3)\ C(6,3)\ D(15,1)$
    $\boxed{0,3}$

5.a) $(x,y) \rightarrow (x, 3y)$ b) $(x,y) \rightarrow (-x, y)$
   c) $(x,y) \rightarrow (x, -y)$ d) $(x,y) \rightarrow (-\frac{x}{3}, y)$

6 →

7. reflect in x axis → $a = -$
   stretch horizontal $B(-1,2)\ C(1,2)$ $f(x) = -f(\frac{1}{3}x)$
                         $B'(-3,-1)\ C'(3,-1)$
                         $\times 3 \rightarrow \frac{1}{b} = \frac{1}{3}$

8. $(x,y) \rightarrow (2x, \frac{1}{3}y)$ $-8 \le x \le 16$
                                            $-2 \le y \le 4$
9 $(x,y) \rightarrow (\frac{x}{3}, -2y)$ $-4 \le x \le 6$
                                            $6 \le x \le -12$ reverse

10. $(x,y) \rightarrow (-3x, y)$ $x = -4 \rightarrow 12$
    1 → $x = -5 \rightarrow -15$

(1.3) 2 → a) $a=3, b=-, h=-3, k=-2$
       b) $b=\frac{1}{2}, a=-, k=7$
       c) $b=4, h=5, k=-1$
       d) $a=\frac{1}{3}, b=2, a=-$

3 —
4. a) $y = af(5(x-3))$
      vert st, hor comp $\frac{1}{5}$, right 3

   b) $y = \frac{1}{4}f(-(x-7))$
      reflect vert st $\frac{1}{4}$, reflect hor, right 7

4c) $y = f(3(y+4))$

$\uparrow$ hor $\uparrow$ left 4
st. $\frac{1}{3}$

5   $a = \frac{1}{}$ $(x,y) \Rightarrow (5x, -y)$

$\xrightarrow{} \times 5$ $b = \frac{1}{5}$ $(-2,0) \to (-10, 0) \longrightarrow$ must be $(-7, 0)$

$\xrightarrow{} +3$

$h = 5$

6. a) $(x,y) \to (x+6, y-8) \to (-12, 4)$
   b) $(x,y) \to (\frac{x}{6}, 2y) \to (-3, 24)$
   c) $(x,y) \to (x-5, -5y+4) \to (-23, -32)$
   d) $(x,y) \to (2x, -y) \to (-36, -12)$

7. $\to$
$1 \to$

（1.4）

$2 \to$ a) $y = x^2$ b) $y = |x|$ c) $x = 2y - 6$ d) $x = (y+4)^2$

$x = y^2$ $x = |y|$ $\frac{x+6}{2} = y$ $\pm\sqrt{x} - 4 = y$

$\pm\sqrt{x} = y$ $x = y, x = -y$

3.   NOT FN.   NOT FN   FN   NOT FN.

4. a) $x = 3y - 6$ b) $x = \frac{1}{2}y + 5$ c) $x = \frac{1}{3}(y+12)$ d) $x = \frac{8y+12}{4}$

$\frac{x+6}{3} = y$ $2(x-5) = y$ $3x - 12 = y$ $\frac{4x-12}{8} = y$

$\frac{x-3}{2} = y$

5a) D$\to x \in \mathbb{R}$ R$\to y \geq 5$

b) $x = (y-3)^2 + 5$ $x \geq 5$ c) $x \geq 3$ or $x \leq 3$

$\pm\sqrt{x-5} + 3 = y$ $y \in \mathbb{R}$

6. a) $x = (y+4)^2$ b) $x = y^2 - 1$ c) $x = (y-2)^2 + 5$

$\pm\sqrt{x} - 4 = y$ $\pm\sqrt{x+1} = y$ $\pm\sqrt{x-5} + 2 = y$

d) $x = (y-5)^2 - 9$

$\pm\sqrt{x+9} + 5 = y$

7a) $x \geq 0$ b) $x \geq -4$ c) $x \leq 3$ d) $x \geq 0$

$x \leq 0$ $x \leq -4$ $x \geq 3$ $x \leq 0$

8. $y = x \cdot 8$
$x = y - 8$
$x + 8 = y = f^{-1}(x)$

a) $f^{-1}(4) = 4 + 8 = 12$

b) $f^{-1}(-2) = -2 + 8 = 6$

c) $f^{-1}(8) = 8 + 8 = 16$

d) $f^{-1}(0) = 0 + 8 = 8$

9 →

(10.1) **COMBINING FUNCTIONS**

1 a) $\sqrt{x-4} + 12$

b) $\dfrac{2x+7} + \dfrac{5x-11} = 7x - 4$

c) $x^2 - 3x + 2 + x^2 - x + 5 = 2x^2 - 4x + 3$

d) $x^2 + 8x + 10 - 7x + 7 = x^2 + x + 17$

2. a) $3x - 5 + x^2 + 1 = x^2 + 3x - 4$

b) →

c) $x \in \mathbb{R} \quad y \geq -6.25$

3 a) $10 - |x+3|$

b) $2x - 5 - (x+8)$
$2x - 5 - x - 8 = x - 13$

c) $x^2 + x + 2 - 2x^2 + 3x = -x^2 + 4x + 5$

d) $4x - 6 - (x^2 - 4x + 4)$
$4x - 6 - x^2 + 4x - 4 = -x^2 + 8x - 10$

4 a) $x^2 + 2x + 1 - 3x = x^2 - x + 1$

b) →

c) $x \in \mathbb{R} \quad y \geq 0.75$

5. a) $x^2 - 6 + \sqrt{x-2} \qquad x \geq 2, \quad y \geq -2$

b) $\sqrt{x-2} - 2x + 1 \qquad x \geq 2, \quad y \leq -3$

c) $2x - 1 - \sqrt{x-2} \qquad x \geq 2, \quad y \geq 3$

d) $x^2 - 6 + 2x - 1 = x^2 + 2x - 7 \qquad x \in \mathbb{R} \quad y \geq -8$

6. a) $x^2 - 7 + 4x + 5 = x^2 + 4x - 2$
$2^2 + 4(2) - 2 = 10$

b) $x^2 - 7 - 4x - 5 = x^2 - 4x - 12$
$1^2 - 4(1) - 12 = -15$

c) $4x + 5 + x^2 - 7 = x^2 + 4x - 2$
$1^2 + 4 - 2 = 3$

7.

x	(a)	(b)	(c)	(d)
4	1	3	4	
-1	4	2	6	
-4	7	1	5	
-5	8	0	8	

8 →

9. a) $x^2 + 5x - 2 - 3x + 4 \Rightarrow x^2 + 2x + 2 = g(x)$

b) $\sqrt{x-7} + 1 - 3x + 4 \Rightarrow \sqrt{x-7} - 3x + 5 = g(x)$

c) $3x + 5 - 3x + 4 \Rightarrow 9 = g(x)$

d) $2x^2 - 7x + 5x \Rightarrow 2x^2 - 10x + 8 = g(x)$

10 a) $C = 2500 + 14p$

$R^{ev} = 65p$

b) →

c) $2500 + 14p = 65p$

$2500 = 51p$

$50 \approx p$

(10.2) 1. a) $(x+3)(2x-5)$

$= 2x^2 - 5x + 6x - 15$

$= 2x^2 + x - 15$

c) $(\sqrt{x-4})(x+2)$

$x\sqrt{x-4} + 2\sqrt{x-4}$

b) $(2x-3)(3x+1)$

$6x^2 + 2x - 9x - 3$

$= 6x^2 - 7x - 3$

d) $(\sqrt{x+1})(\sqrt{3-y})$

2 —

3 a) $\dfrac{x+3}{2x-5}$ $x \neq \frac{5}{2}$

b) $\dfrac{2x-3}{3x+1}$ $x \neq \frac{-1}{3}$

c) $\dfrac{\sqrt{x-4}}{x+2}$ $x \neq 2$ $x \geq 4$

d) $\dfrac{\sqrt{x+1}}{\sqrt{3-y}}$ $x \geq -1$ $x \leq 3$

$y \neq \dfrac{\frac{x}{x} + \frac{3}{x}}{\frac{2x}{x} - \frac{5}{x}} = \dfrac{1+0}{2-0} = \dfrac{1}{2}$

$y \neq \dfrac{\frac{2x}{x} - \frac{3}{x}}{\frac{3x}{x} + \frac{1}{x}} = \dfrac{2-0}{3+1} = \dfrac{2}{3}$

$0 \leq y \leq 0.2$ approx

$0 \leq y \leq 5$ approx

4 —

5. a) $(x+1)(2x-1) = 2x^2 + 3x + 1$ $x \in R$ $y \geq -0.125$

b) $(x+1)(2x^2 + 7x + 3) = 2x^3 + 7x^2 + 3x$

$2x^2 + 7x + 3$

$\overline{2x^3 + 9x^2 + 10x + 3}$ $x \in R$ $y \in R$

c) $\dfrac{2x+1}{x+1}$ $x \neq -1$ $y \neq 2$

d) $\dfrac{(2x+1)(x+2)}{2x+1}$ $x \neq -\frac{1}{2}$ $y \neq 2\frac{1}{2}$

6. a) $(x^2 + 4x + 3)(x - 5) = x^3 + 4x^2 + 3x$
$$\underline{\quad\quad -5y^2 - 20x - 15}$$
$$x^3 - x^2 - 17x - 15 \qquad x \in \mathbb{R} \quad y \in \mathbb{R}$$

b) $(x - 4)(x^2 - 16) = x^3 - 16x - 4x^2 + 64 \qquad x \in \mathbb{R} \quad y \in \mathbb{R}$

c) $\dfrac{1}{(x-3)} \cdot \dfrac{1}{(x+1)} \qquad \begin{array}{l} x \neq 3 \\ x \neq -1 \end{array} \qquad y \neq 0$

7.

x	f(x)	g(x)	f·g	f/g	g/f
-1	4	3	12 (a)		
2	1	-3	-3 (b)	-4 (c)	
1	4	-1		4/5 (d)	1/5 (e)
3	-4	-5			5 (e)
0	5	1			
-2	1	5			

8. —

9. a) $\dfrac{f(x)}{h(x)} = g(x)$

$$x^2 + 3x + 2 \ )\overline{\ x^3 + 6x^2 + 11x + 6\ } \quad \boxed{x + 3} \ g(x)$$
$$\underline{x^3 + 3x^2 + 2x}$$
$$3x^2 + 9x + 6$$
$$\underline{3x^2 + 9x + 6}$$
$$0$$

b) —

10. a) $(x+1)(x-5)(x-4)$
$= (x^2 - 4x - 5)(x - 4)$
$x^3 - 4x^2 - 5x$
$\underline{-4x^2 + 16x + 20}$
$x^3 - 8x^2 + 11x + 20$

b) $\dfrac{(x+1)(x-5)}{x-4} = \dfrac{x^2 - 4x - 5}{x-4}$

c) $\dfrac{2x - 4}{x - 4}$

(10.3)

1. a) $g(1) = -2(1) + 7 = 5 \qquad f(5) = 3(5) - 5 = 10$

b) $g(-4) = -2(-4) + 7 = 15 \qquad f(15) = 3(15) - 5 = 40$

c) $f(-2) = 3(-2) - 5 = -11 \qquad g(-11) = -2(-11) + 7 = 29$

d) $f(3) = 3(3) - 5 = 4 \qquad g(4) = -2(4) + 7 = -1$

2.a) $g(1) = 1^2 + 3(1) + 4 = 8$ $f(8) = 8 - 2(8) = -8$

b) $g(-4) = (-4)^2 + 3(-4) + 4 = 8$ $f(8) = -8$

c) $f(-2) = 8 - 2(-2) = 12$ $g(12) = 12^2 + 3(12) + 4 = 184$

d) $f(3) = 8 - 2(3) = 2$ $g(2) = 2^2 + 3(2) + 4 = 14$

3.a) $g(7) = 1$ $f(-1) = 7$

b) $g(4) = 3$ $f(3) = 3$

c) $f(1) = 1$ $g(7) = 1$

d) $f(7) = 5$ $g(5) = -2$

4.a) $f(-1) = 4$ $g(4) = 4$ b) $f(-2) = 5$ $g(5) = 6$

c) $g(1) = -2$ $f(-2) = 5$ d) $g(2) = 0$ $f(0) = 3$

5.a) $g(a) = a^2 + 6$ $f(g(a)) = 2(a^2 + 6) - 9 = 2a^2 + 12 - 9 = 2a^2 + 3$

b) $f(a) = 2a - 9$ $g(f(a)) = (2a - 9)^2 + 6 = 4a^2 - 36a + 81 + 6 = 4a^2 - 36a + 87$

c) $f(g(x)) = 2(x^2 + 6) - 9 = 2x^2 + 12 - 9 = 2x^2 + 3$

d) $g(f(x)) = (2x - 9)^2 + 6 = (b)$

e) $f(f(x)) = 2(2x - 9) - 9 = 4x - 18 - 9 = 4x - 27$

f) $g(g(x)) = (x^2 + 6)^2 + 6 = x^4 + 12x^2 + \ldots$

6.a) $f(g(x)) = (\sqrt{x + 2})^2 - 5 = x + 2 - 5 = x - 3$

b) —

(c) $x \geq -2$ $y \geq -5$

7   $f(g(x)) = \sqrt{x + 5 + 2} = \sqrt{x + 1}$ $x \geq -7$ $y \geq 0$

$g(f(x)) = \sqrt{x + 2} + 5$ $x \geq -2$ $y \geq 5$

8 —

9.a) $f(g(x)) = (\uparrow)^2 = x - 4$
$\quad g(x) = \sqrt{x - 4}$

b) $f(g(x)) = (\uparrow)^2 - 4 = x^2 + 6x + 5$
$\quad g(x) = x + 3$

Exponents Review

1 —   2 —   3 —   4 —

5.a) $9 = c^{-1}$ $3 = c^{-1}$ $y = (\frac{1}{3})^x$
                    $3^2 = c^{-2}$ $3^1 = c^{-1}$
                                            $= c^{-1}$
                                            $= \frac{1}{c}$

b) $6 = c^1$ $36 = c^2$ $y = 6^x$

c) $4 = c^2$ $y = 2^x$
    2^2

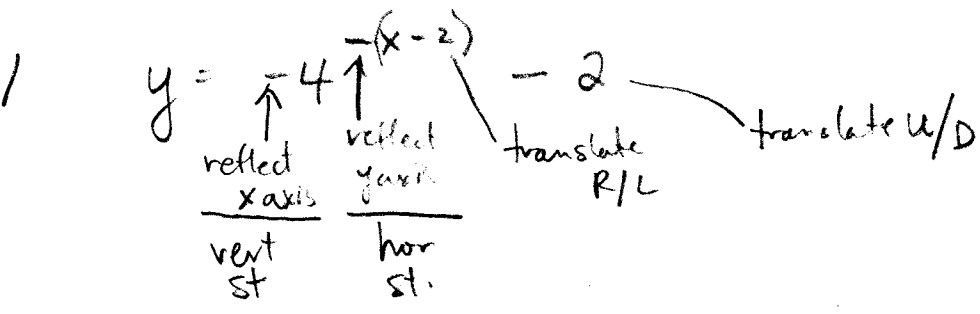d) $25 = c^{-2}$ $y = (\frac{1}{5})^x$
    $5^2 = \frac{1}{c^2}$

6. $P = 100 \, e^{-0.139(5)} = 50 \, kPa$

7. a) $20\% \rightarrow 100 - w = 80\%$ left $\rightarrow 0.80$
   c) $t = 5$ subst.

8. $\frac{1}{2}$ life $\rightarrow 0.5$ $a = 100$
    $t = $ time $\rightarrow \frac{t}{13}$

9 —   10 —

1
   $y = -4^{-(x-2)} - 2$
   reflect x axis — vert st
   reflect y axis — hor st.
   translate R/L
   translate U/D

2. a) decreasing graph — (0,1) moved right 2
   b) decreasing graph that is reflected in y axis, stretched 2 $\rightarrow (0,1) \Rightarrow (0,2)$ vert
   c)   "   "   "   "   "   "   "   x axis, hor. st. $\frac{1}{2} \rightarrow (0,1) \Rightarrow (0,1)$
   d)   "   "   ", hor st 2 $\rightarrow (0,1) \rightarrow (0,1)$

3.

$y=5^x$	$y=-5^x$	$y=-\frac{1}{2}(5)^x$	$-\frac{1}{2}(5)^{x+4}-6$
(x,y)	$(x,-y)$	$(x,-\frac{1}{2}y)$	$(x-4,-\frac{1}{2}y-6)$

4. a) $a=4$ vert st. $b=1$ no change $h=0$ no change $K=6$ up
   b) $a=-1$ reflect in x $b=1$ " $h=4$ right $k=0$ no change
   c) $a=3/2$ vert st. $b=4$ horst $\frac{1}{4}$ $h=9$ right $k=-8$ down
   d) $a=\frac{1}{2}$ " " $b=-\frac{4}{5}$ " " $5/4$ +reflect^n_y $h=-2$ L $k=\frac{7}{4}$ u

5. a) $(x,y) \rightarrow (2x+1, y+7)$
   b) $(x,y) \rightarrow (\frac{1}{2}x-5, -3y)$
   c) $(x,y) \rightarrow (x+6, \frac{1}{2}y-5)$

6. a) vert st $\rightarrow a=3$, reflect y $\rightarrow b=-$, $4L$ $h=-4$, $3D$ $k=-3$
   b) hor st $\rightarrow b=2$, " x $\rightarrow a=-$, $7u$ $k=7$
   c) $a=-4$ $b=2$ $h=3$ $k=5$
   d) $a=\frac{1}{3}$ $b=1$ $h=-1$ $k=0$

7. —

8. a) $a=0.5 \rightarrow$ vert st. , $b=-2 \rightarrow$ reflect x^n, horst $\frac{1}{2}$, $h=-4$ L, $k=7$ u
   b)
   c) $(x,y) \rightarrow (-\frac{1}{2}x-4, 0.5y+7)$

9.

years after 2011	population
0	35000
1	$35000+35000(0.024)$
	$35000(1+0.024)$
	$35000(1.024)$
2	$1.024(35000(1.024))$
3	$1.024(1.024(35000(1.024)))$
x	$(1.024)^x(35000)$

a) ——————
b) → c) $x=5$ pop $=39,406$

10

mile	pressure
0	14.7
1	$0.08(14.7)$
2	$0.08(0.08(14.7))$
m	$0.08^m(14.7)$

$(100-20\% = 80\% = 0.08)$

b) $\quad 0.08^5(14.7) = 5\ lb/in^2$

1. a) $0.5 = \dfrac{1}{2} = 2^{-1}$

b) $32^{\frac{1}{2}} = (2^5)^{\frac{1}{2}} = 2^{\frac{5}{2}}$

c) $512 = 2^9$

d) $\left(\dfrac{1}{16}\right)^5 = \left(\dfrac{1}{2^4}\right)^5 = (2^{-4})^5 = 2^{-20}$

2. a) $25 = 5^2$

$\dfrac{1}{125} = \dfrac{1}{5^3} = 5^{-3}$

b) $27 = 3^3$

$\sqrt[3]{81} = (81)^{\frac{1}{3}} = (3^4)^{\frac{1}{3}} = 3^{4/3}$

c) $0.25 = \dfrac{1}{4} = \dfrac{1}{2^2} = 2^{-2}$

$8 = 2^3$

d) $6^{\frac{1}{3}}$

$6^2 \, 6^{\frac{1}{2}} = 6^{2\frac{1}{2}}$

3. a) $3^{4x}(3) = 27^{2x}$

$3^{4x+1} = 3^{6x}$

$4x+1 = 6x$

$1 = 2x$

$\dfrac{1}{2} = x$

b) $\left(\dfrac{4}{7}\right)^{5x} = \left(\dfrac{4^3}{7^3}\right)^{2x-1}$

$= \left(\dfrac{4}{7}\right)^{6x-3}$

$5x = 6x-3$

$3 = x$

c) $3^{-2x} = \dfrac{3^{3x}}{3^{4x-2}}$

$-2x = 3x - (4x-2)$

$-2x = 3x - 4x + 2$

$-x = 2$

$x = -2$

d) $2^{x-1} = (2^{7x})2^x$

$2^{x-1} = 2^{8x}$

$x-1 = 8x$

$-1 = 7x$

$-\dfrac{1}{7} = x$

4. a) $2^{4x+4} = 2^{3-3x}$

$4x+4 = 3-3x$

$7x = -1$

$x = -\dfrac{1}{7}$

b) $3^{3x+6} = 3^{-3+6y}$

$3x+6 = -3+6y$

$9 = 3x$

$3 = x$

4c) $2^{3x-3} = 2^{-20+4x}$

$3x-3 = -20+4x$

$17 = x$

d) $6^{-3x+2} = 6^{2x+8}$

$-3x+2 = 2x+8$

$-6 = 5x$

$-6/5 = x$

5a) $\dfrac{800}{500} = 1.03^t$

$1.6 = 1.03^t$

$1.03^{\boxed{16}} = 1.604$

b) $\dfrac{5}{200} = (\frac{1}{2})^{3t}$

$\sqrt[3]{0.025} = (\frac{1}{2})^{3t}$

$0.292 = \frac{1}{2}^t$

$0.5^2 = 0.25$

$0.5^{1.75} = 0.297$

c) $3^t = 2^{t+4}$

$t=1$	3	32
$t=4$	81	256
$t=8$	6561	4096
$t=6$	729	1024
$t=6.5$	1262.7	1448.2

d—

6a) $A = 3000\left(1 + \dfrac{0.052}{2}\right)^{2t}$

b) $2500\left(1 + \dfrac{0.04}{4}\right)^{4t}$

c) $8000\left(1 + \dfrac{0.06}{12}\right)^{12t}$

d) $6800(1 + 0.021)^t$

7. $8000 = 5000\left(1 + \dfrac{0.072}{12}\right)^{12t}$

$1.6 = (1.006)^{12t}$

$1.0399\ldots = (1.006)^t$

$t=6 \rightarrow 1.0365\ldots$

$t=6.5 \rightarrow 1.0396\ldots$

8. a) $3500\left(1 + \dfrac{0.05}{2}\right)^8 = \$4264.41$

b) $7000 = 3500\left(1 + \dfrac{0.05}{2}\right)^{4t}$

$2 = (1.025)^{4t}$

$1.189\ldots = 1.025^t \qquad t \sim 14$