In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes = 7.08×10^3 s and orbital speed of 3.40×10^3 m/s . The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

a) Calculate the radius of the GS orbit.

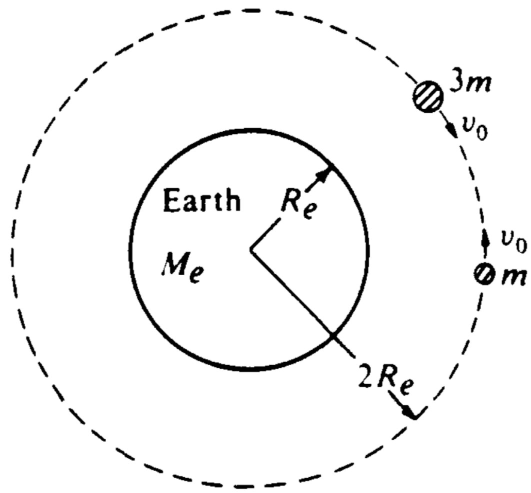
b) Calculate the mass of Mars.

c) Calculate the total mechanical energy of the GS in this orbit.

d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period? Justify your answer.

An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^{7}$ m. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^{4} s. Determine the required orbital radius in meters.

Two satellites, of masses m and $3m$, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_0 but in opposite directions.



a) Calculate the orbital speed v_0 of the satellites in terms of G, M_e, and R_e.

b) Assume that the satellites collide head-on and stick together. In terms of v_0 find the speed v of the combination immediately after the collision.

c) Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.
b) Use your expression from part a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.
c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's
mass?
d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.
e) Plot the graph. Label the axes with the variables used and appropriate numbers to indicate the scale.

f) Using the graph, calculate a value for the mass of Saturn.

In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18 × 10^2 minutes = 7.08 × 10^3 s and orbital speed of 3.40 × 10^3 m/s . The mass of the GS is 930 kg and the radius of Mars is 3.43 × 10^6 m.

a) Calculate the radius of the GS orbit.

b) Calculate the mass of Mars.

c) Calculate the total mechanical energy of the GS in this orbit.

d) If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period? Justify your answer.

$T = 7.08 \times 10^3$ s

$m = 930$ kg

$R_M = 3.43 \times 10^6$ m

$v = 3.40 \times 10^3$ m/s

a) $v = \frac{d}{t} = \frac{2\pi r}{T}$

$r = \frac{vT}{2\pi} = \boxed{3.83 \times 10^6 \text{ m}}$

b) $F_c = ma_c$

$F_g = ma_c$

$G\frac{Mm}{r^2} = m\frac{v^2}{r}$

$M = \frac{v^2 r}{G} = \boxed{6.64 \times 10^{23} \text{ kg}}$

c) $E_T = E_K + E_P$

$= \frac{1}{2}mv^2 - G\frac{Mm}{r}$

$= \boxed{-5.375 \times 10^9 \text{ J}}$

d) LOWER

According to Keplers Law of Periods

$T^2 \propto R^3$ if $R\downarrow, T\downarrow$

An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass *M_J* = 1.90 × 10^27 kg and radius *R_J* = 7.14 × 10^7 m. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55 × 10^4 s. Determine the required orbital radius in meters.

$M_J = 1.90 \times 10^{27}$ kg

$R_J = 7.14 \times 10^{7}$ m

$T = 3.55 \times 10^{4}$ s

$r = ?$

$F_c = ma_c$

$F_g = ma_c$

$$G\frac{M\cancel{m}}{r^2} = \cancel{m}\frac{4\pi^2 r}{T^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

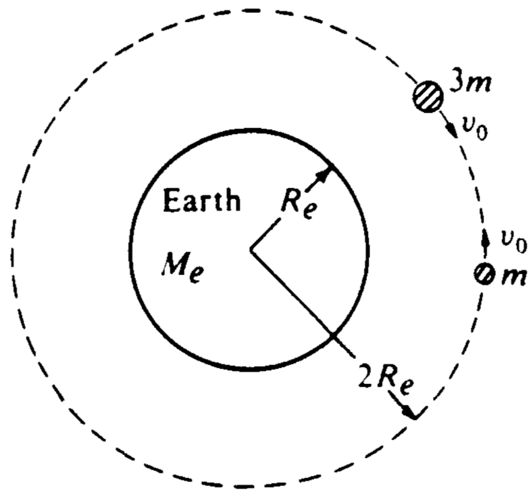
$$= \boxed{1.59 \times 10^{8}\ \text{m}}$$

Two satellites, of masses *m* and 3*m*, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass *M_e* and radius *R_e*. In this orbit, which has a radius of 2*R_e*, the satellites initially move with the same orbital speed *v_0* but in opposite directions.

a) Calculate the orbital speed *v_0* of the satellites in terms of *G*, *M_e*, and *R_e*.

b) Assume that the satellites collide head-on and stick together. In terms of *v_0* find the speed *v* of the combination immediately after the collision.

c) Calculate the total mechanical energy of the system immediately after the collision in terms of *G*, *m*, *M_e*, and *R_e*. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.



a) $F_c = ma_c$

$F_g = ma_c$

$G\frac{M\cancel{m}}{r^2} = \cancel{m}\frac{v^2}{r}$

$v = \sqrt{\frac{GM}{r}} = \boxed{\sqrt{\frac{GM_e}{2R_e}}}$

b) $p_i = p_f$

$p_{1i} + p_{2i} = p_f$

$m_1v_{1i} + m_2v_{2i} = Mv_f$

$3\cancel{m}v_0 - \cancel{m}v_0 = 4\cancel{m}v$

$\boxed{v = \frac{1}{2}v_0}$

$= \frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}$

c) $E_T = E_k + E_p$

$= \frac{1}{2}mv^2 - G\frac{Mm}{r}$

$= \frac{1}{2}(4m)\left(\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}\right)^2 - G\frac{M_e(4m)}{2R_e}$

$= \frac{1}{2}(4m)\left(\frac{1}{8}\frac{GM_e}{R_e}\right) - 2G\frac{M_em}{R_e} = \boxed{-\frac{7}{4}G\frac{M_em}{R_e}}$

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass *M_S* of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	$R^{3/2}$ $(m^{3/2})$	
8.14×10^4	1.85×10^8	2.52×10^{12}	
1.18×10^5	2.38×10^8	3.67×10^{12}	
1.63×10^5	2.95×10^8	5.07×10^{12}	
2.37×10^5	3.77×10^8	7.32×10^{12}	

a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

b) Use your expression from part a) and the assumption of circular orbits to derive an equation for the orbital period *T* of a moon as a function of its orbital radius *R*.

c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's
mass?

d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

e) Plot the graph. Label the axes with the variables used and appropriate numbers to indicate the scale.

f) Using the graph, calculate a value for the mass of Saturn.

a) $F_g = G\frac{Mm}{r^2} = \boxed{G\frac{M_s m}{R^2}}$

b) $F_c = ma_c$

$F_g = ma_c$

$$G\frac{M_s \cancel{m}}{R^2} = \cancel{m}\frac{4\pi^2 R}{T^2}$$

$$\boxed{T = \sqrt{\frac{4\pi^2 R^3}{GM_s}}}$$

$$= \sqrt{\frac{4\pi^2}{GM_s}}\, R^{3/2}$$

c) T vs. $R^{3/2}$

or

T^2 vs. R^3

f) $T = k R^{3/2}$

$k =$ slope

$= \dfrac{\Delta T}{\Delta R^{3/2}}$

$= 3.24 \times 10^{8} \ \frac{s}{m^{3/2}}$

$$T = \boxed{\sqrt{\frac{4\pi^2}{G M_s}}} R^{3/2}$$

$$\underset{k}{\shortparallel}$$

$$k = \sqrt{\frac{4\pi^2}{G M_s}}$$

$$M_s = \frac{4\pi^2}{G k^2}$$

$$= \boxed{5.64 \times 10^{26} \ kg}$$