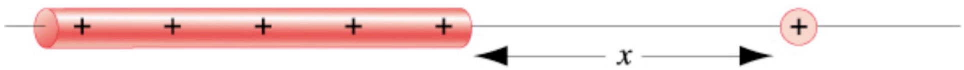
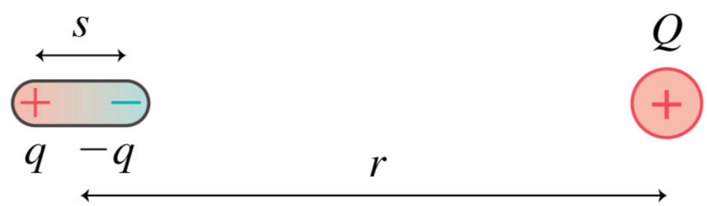
a) Find the force on a positive point charge q located a distance x from the end of a rod of length L with uniformly distributed positive charge Q (shown below). *Requires calculus.*



b) Now consider the limiting case, where the rod is far, far away, compared to the length of the rod (i.e. $x >> L$). What is the force in this limit?

Consider the electric dipole with charges +*q* and –*q* separated by the fixed distance *s*, as shown. A charge +*Q* is distance *r* from the center of the dipole.

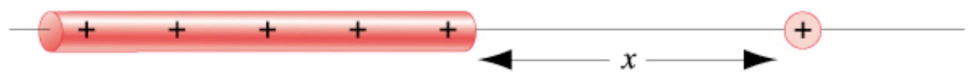


a) Is this force towards or away from the charge +*Q*? Explain, in one sentence.

b) Write an expression for the net force exerted on the dipole by charge +*Q*.

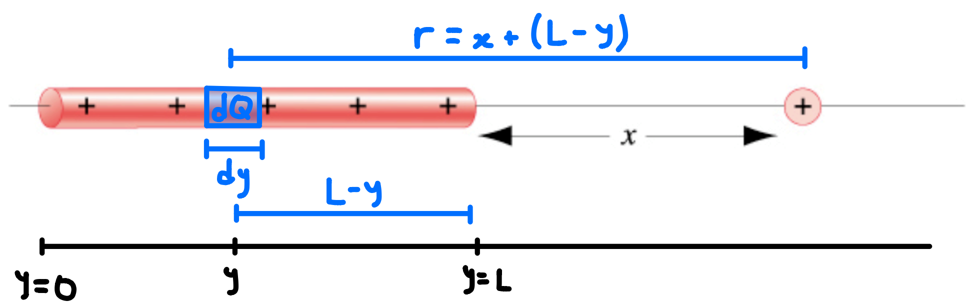
c) Use the binomial approximation $(1+x)^{-n} \approx 1 - nx$ for $x \ll 1$ to show that your result in part b) may be written as $F_{net} = k\dfrac{2qQs}{r^3}$

a) Find the force on a positive point charge q located a distance x from the end of a rod of length L with uniformly distributed positive charge Q (shown below). *Requires calculus.*



b) Now consider the limiting case, where the rod is far, far away, compared to the length of the rod (i.e. $x >> L$). What is the force in this limit?

Divide the rod into a series of infinitesimal lengths, dy, each with charge $dQ=(Q/L)\,dy$. (We're using the variable y here because x is the fixed (constant) distance between the charge and the rod.)
We see that each infinitesimal length contributes to the net force only the horizontal direction to the right, so we need to sum up these horizontal components only. The other two components of the force are zero.
Take the left edge of the rod to be y=0. With this origin, the right edge of the rod is at $y=L$, and the charge q is at $y=L+x$. The distance between the point charge q and each infinitesimal bit of the rod will therefore be $x+(L-y)$.



$$F = \int dF = \int_{y=0}^{y=L} k\frac{q\,dQ}{r^2}$$

$$= \int_{y=0}^{y=L} k\frac{q\left(\frac{Q}{L}\right)dy}{(x+L-y)^2}$$

$$= k\frac{qQ}{L}\int_{y=0}^{y=L}\frac{dy}{(x+L-y)^2}$$

$u = x+L-y$
$du = -dy$

$$= k\frac{qQ}{L}\int_{y=0}^{y=L} -\frac{du}{u^2}$$

$$= k\frac{qQ}{L}\left[\frac{1}{u}\right]_{y=0}^{y=L}$$

$$= k\frac{qQ}{L}\left[\frac{1}{x+L-y}\right]_{y=0}^{y=L}$$

$$= k\frac{qQ}{L}\left(\frac{1}{x} - \frac{1}{x+L}\right)$$

$$= k\frac{qQ}{L}\left(\frac{x+L}{x(x+L)} - \frac{x}{x(x+L)}\right)$$

$$= k\frac{qQ}{\cancel{L}}\left(\frac{\cancel{L}}{x(x+L)}\right)$$

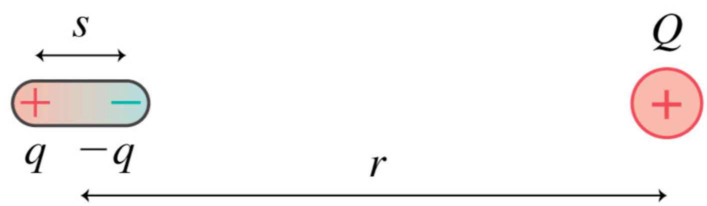
$$= \boxed{k\frac{qQ}{x(x+L)} \quad \text{RIGHT}}$$

b) FAR AWAY FROM THE ROD $(x >> L)$

$$\boxed{F = k\frac{qQ}{x^2}}$$

THE ROD "LOOKS" LIKE A POINT CHARGE FROM FAR AWAY.

Consider the electric dipole with charges $+q$ and $-q$ separated by the fixed distance s, as shown. A charge $+Q$ is distance r from the center of the dipole.



a) Is this force towards or away from the charge +Q? Explain, in one sentence.
b) Write an expression for the net force exerted on the dipole by charge +Q.

c) Use the binomial approximation $(1+x)^{-n} \approx 1 - nx$ for $x \ll 1$ to show that your result in part b) may be written as $F_{net} = k\dfrac{2qQs}{r^3}$

a) TOWARDS THE CHARGE Q BECAUSE THE ATTRACTIVE FORCE DUE TO Q ON THE NEGATIVE CHARGE ON THE DIPOLE IS GREATER THAN THE REPULSIVE FORCE OF Q ON THE POSITIVE CHARGE.

b) CHARGE Q AND THE DIPOLE CHARGES (q AND $-q$) ARE POINT CHARGES.
THE FORCE ON THE DIPOLE IS THE VECTOR SUM OF THE FORCES ON q AND $-q$.

$$\vec{F}_{Q \text{ on } +q} = k\frac{Qq}{(r+\frac{s}{2})^2} \quad \text{AWAY FROM } Q$$

$$\vec{F}_{Q \text{ on } -q} = k\frac{Qq}{(r-\frac{s}{2})^2} \quad \text{TOWARDS } Q$$

$$\boxed{\vec{F}_{NET} = kQq\left(\frac{1}{(r-\frac{s}{2})^2} - \frac{1}{(r+\frac{s}{2})^2}\right) \text{ TOWARDS } Q}$$

c) $$F_{NET} = kQq\left(\frac{1}{(r-\frac{s}{2})^2} - \frac{1}{(r+\frac{s}{2})^2}\right)$$

$$= kQq\left(\frac{1}{r^2(1-\frac{s}{2r})^2} - \frac{1}{r^2(1+\frac{s}{2r})^2}\right)$$

$$= k\frac{Qq}{r^2}\left(\left(1-\frac{s}{2r}\right)^{-2} - \left(1+\frac{s}{2r}\right)^{-2}\right)$$

IF $\frac{s}{2r} \ll 1$, $\left(1 \pm \frac{s}{2r}\right)^{-2} \approx 1 \mp \frac{2s}{2r} = 1 \mp \frac{s}{r}$

$$= k\frac{Qq}{r^2}\left(\left(1+\frac{s}{r}\right) - \left(1-\frac{s}{r}\right)\right)$$

$$= k\frac{Qq}{r^2}\left(\frac{2s}{r}\right)$$

$$= k\frac{2Qqs}{r^3}$$

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