SOLVING RADICAL EQUATIONS

Use the same strategies for solving linear equations to solve radical equations.

You need to isolate the variable by doing the inverse operations.

ex

$13 = 2\sqrt{x+1} - 7$
$+7 \qquad\qquad +7$

$\frac{20}{2} = \frac{\cancel{2}\sqrt{x+1}}{\cancel{2}}$

$(10)^2 = (\sqrt{x+1})^2$

$100 = x+1$
$-1 \qquad -1$

$x = 99$

Define the variable
$x \geq -1$

$x+1 \geq 0$
$-1 \quad -1$

$x \geq -1$

★ Check is $x \geq -1$
Put the solution into the original equation

Check $13 = 2\sqrt{99+1} - 7$

$13 = 2\sqrt{100} - 7$

$13 = 2(10) - 7$

$13 = 13$ ✓

The solution to a radical equation is a root of the equation or REAL ROOT.

Sometimes, your solution or answer will not satisfy the original equation. The value is an EXTRANEOUS root.
It is a root of the equation formed by squaring the radical but is not a root of the ORIGINAL equation.

Ex. $\sqrt{2x-5} = \sqrt{x-7}$ ← Define variables $2x-5 \geq 0$, $2x \geq 5$, $x \geq 5/2$; $x-7 \geq 0$, $x \geq 7$

$(\sqrt{2x-5})^2 = (\sqrt{x-7})^2$

$2x - 5 = x - 7$
$-x \qquad -x$

$x - 5 = -7$
$+5 \qquad +5$

$x = -2$

extraneous solution or root
not a real root

Check

$\sqrt{2x-5} = \sqrt{x-7}$

$\sqrt{2(-2)-5} = \sqrt{-2-7}$

$\sqrt{-9} = \sqrt{-9}$ not possible

P.142
CYU

2) $4\sqrt{x} + 3 = 5\sqrt{x} + 1$ $\qquad x \geq 0$
$-4\sqrt{x} \qquad\qquad -4\sqrt{x}$

$3 = \sqrt{x} + 1$
$-1 \qquad\qquad -1$

$(2)^2 = (\sqrt{x})^2$

$4 = x$

$4\sqrt{4} + 3 = 5\sqrt{4} + 1$

$4(2) + 3 = 5(2) + 1$

$8 + 3 = 10 + 1$

$11 = 11$ ✓

Applications of radical equations

Ex. Investigators can approximate the initial velocity of a car based on the length of a car's skid. (L in metres)
The formula models the relationship

$\boxed{V = 12.6\sqrt{L} + 8}$ $\quad L \geq 0$

What length of skid is expected if a car is travelling 50km/h when the brakes are applied?

$$\frac{50}{-8} = 12.6\sqrt{L} \quad \frac{+8}{-8}$$

$$\frac{42}{12.6} = \frac{12.6\sqrt{L}}{12.6}$$

$$\left(3\tfrac{1}{3}\right)^2 = \left(\sqrt{L}\right)^2$$

$$11.1 = L$$

The mass (kg) that a beam (cm) with fixed width and length can support is related to its thickness.

The formula is $t = \frac{1}{5}\sqrt{\frac{m}{3}}$ $\quad m \geq 0$

If the beam is 4cm, what mass can it support?

$$(5)\ 4 = (5)\tfrac{1}{5}\sqrt{\tfrac{m}{3}} \qquad 3 \times 400 = \tfrac{m}{3} \times 3$$

$$(20)^2 = \left(\sqrt{\tfrac{m}{3}}\right)^2 \qquad 1200 = m$$

p. 145 4-7, 8, 9, 10, 12

b) $s^2 = \left(20\sqrt{273 + 20}\right)^2$

$s = 20\sqrt{293}$ $\qquad \sqrt{289} = 17$

$s = \underbrace{20\,(17.1)}$

$s =$

→ Extraneous root

$\sqrt{-9} = \sqrt{-9}$

Test

5 - non-calc Q

Calc section
10 - mc
5 - short answer/word problem

p 156 - 161 - all

p. 164 #6, 7

$x = -4$

$$\sqrt{8-7x} \geq 0$$

$$8 - 7x \geq 0$$

$$\frac{-8 \qquad -8}{}$$

$$\frac{-7x}{-7} \geq \frac{-8}{-7}$$

No $x \leq \frac{8}{7}$ $\qquad x \geq \frac{8}{7}$

$$\sqrt{-9} = \sqrt{-9}$$

4)

$$(3x)^2 = \left(\sqrt{30+3x}\right)^2 \qquad a^2+2ab+b^2 = (a+b)^2$$

$$9x^2 = 30 + 3x$$

$$9x^2 - 3x - 30 = 0$$

$$3(3x^2 - 1x - 10) = 0$$

$$3(\quad x \quad) = 0$$

$$3x^2 - 1x - 10 \qquad \begin{matrix} -30 \\ +5 \; -6 \end{matrix}$$

$$(3x^2 - 6x) + (5x - 10)$$

$$3x(x-2) + 5(x-2)$$

$$3(x-2)(3x+5) = 0$$

$$x = 2 \text{ or } -5/3$$

17 a) $(4)^3 = \left(\sqrt[3]{8x}\right)^3$

$$\frac{64}{8} = \frac{8x}{8}$$

$$8 = x$$

b) $\left(\sqrt[3]{2x-5}\right)^3 = (3)^3 \qquad \cancel{x \neq \frac{5}{2}} \quad x \in \mathbb{R}$

$$2x - 5 = 27$$

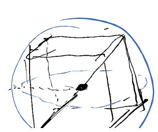
$$2x = 32$$

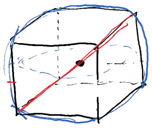
$$x = 16$$

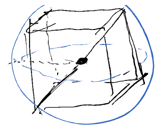
Chapter Review

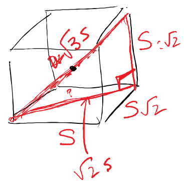
2, 3, 4, 5, 6, 7, 8 (practise), 9

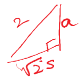
11 – 13











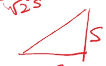
$2^2 - (\sqrt{2}s)^2 = a^2$

$4 - 2s^2 =$

$\frac{-2s^2}{-2} = \frac{-4}{-2}$

$\sqrt{s^2} = \sqrt{2}$

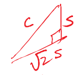
$s = \sqrt{2}$



$s^2 + s^2 = c^2$

$\sqrt{2s^2} = \sqrt{c^2}$

$\sqrt{2}s = c$



$(\sqrt{2}s)^2 + s^2 = c^2$

$2s^2 + s^2 = c^2$

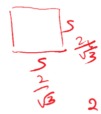
$\sqrt{3s^2} = \sqrt{c^2}$

$\sqrt{3}s = c$

$\sqrt{3}s = \text{diameter} = 2$

$\frac{\sqrt{3}s}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$s = \frac{2}{\sqrt{3}}$



$\frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{4}{3} \times 6 = \frac{24}{3} = 8$