

$$\begin{aligned} P &= 100 & \text{max Area} \\ 2x + 2y &= 100 \\ x + y &= 50 \\ y &= 50 - x \end{aligned}$$

$$A = x(50-x) = 50x - x^2$$

$$\frac{dA}{dx} = 50 - 2x = 0 \\ x = 25$$

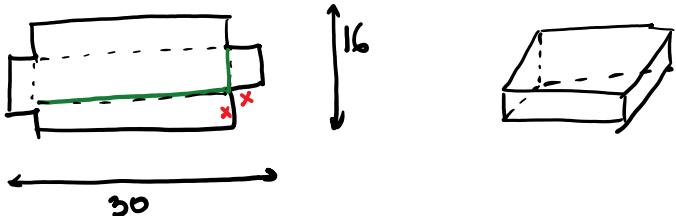
x	0	25		50
$A'(x)$	+	+	0	---
$A(x)$	0	↗	625	0

$$\text{max } A = 625 \text{ ft}^2 \rightarrow \boxed{\begin{array}{|c|c|} \hline & 25 \\ \hline 25 & \\ \hline \end{array}}$$

$$A'' = -2$$

$$A''(x=25) = -2 < 0 \Rightarrow A \text{ is max at } x=25$$

(2)



$$V = (30-2x)(16-2x) \cdot x \quad \text{maximum}$$

$$V = 4x^3 - 92x^2 + 480x \quad 0 \leq x \leq 8$$

$$V' = 12x^2 - 184x + 480 = 0$$

$$4(3x^2 - 46x + 120) = 0 \\ x = \frac{46 \pm \sqrt{(-46)^2 - 4 \cdot 3 \cdot 120}}{6} \quad \left\{ \begin{array}{l} 12x \\ \frac{10}{3} \end{array} \right.$$

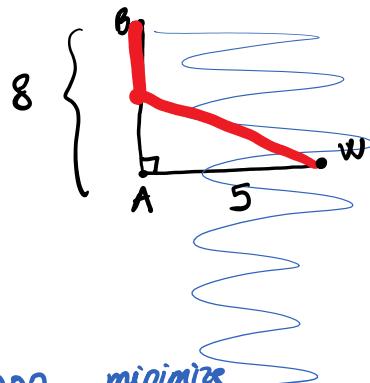
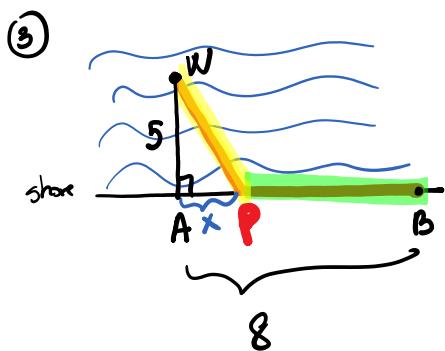
$$V'' = 6x - 46$$

$$V'' \Big|_{x=12} = 6 \cdot 12 - 46 > 0 \\ V'' \Big|_{x=\frac{10}{3}} = 6 \cdot \frac{10}{3} - 46 < 0$$

$$\text{max } V \text{ occurs when } x = \frac{10}{3}$$

$$1x = -3$$

$$V\left(\frac{10}{3}\right) \approx 7.6 \text{ in}^3$$



$$C = wP \cdot 100000 + P_B \cdot 75000$$

$$\sqrt{x^2 + 25}$$

$$8-x$$

minimize

$$C = 100000\sqrt{x^2 + 25} + 75000(8-x)$$

$$C' = 100000 \cdot \frac{1}{2\sqrt{x^2 + 25}} \cdot 2x - 75000$$

$$= \frac{100000x}{\sqrt{x^2 + 25}} - 75000 = 0$$

$$\frac{100000x}{\sqrt{x^2 + 25}} = 75000$$

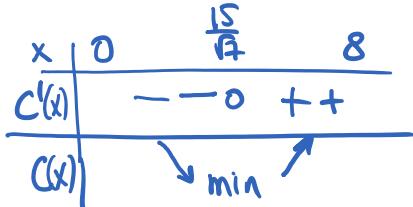
$$\frac{4x}{\sqrt{x^2 + 25}} = 3$$

$$16x^2 = 9(x^2 + 25)$$

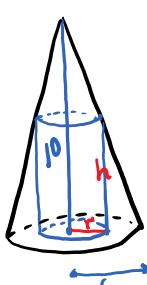
$$7x^2 = 225$$

$$x = \pm \frac{15}{\sqrt{7}} \text{ reject } -\frac{15}{\sqrt{7}}$$

$$x = \frac{15}{\sqrt{7}}$$



Point P is $\frac{15}{4} \approx 5.6 \text{ mi away from A}$



$$V_{\text{cylinder}} = \max$$

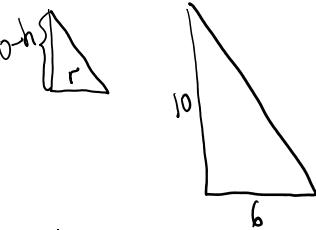
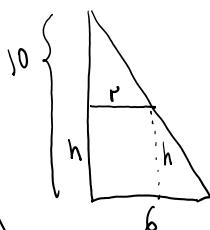
$$V = \pi r^2 h$$

$$V = \pi r^2 (10 - \frac{5r}{3})$$

$$0 < r < 6$$

$$V = 10\pi r^2 - \frac{5}{3}\pi r^3$$

$$V = 20\pi r - 5\pi r^2 = 0$$



$$\frac{10-h}{10} = \frac{r}{6}$$

$$60 - 6h = 10r$$

$$6h = 60 - 10r$$

$$h = 10 - \frac{5r}{3}$$

$0 < r < 6$

$$V = 20\pi r - 5\pi r^2 = 0$$

$$5\pi r(4-r) = 0$$

$$r=0 \text{ or } r=4$$

$$60 - 6\pi h = 10\pi$$

$$6h = 60 - 10\pi$$

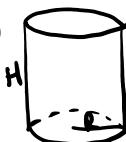
$$h = 10 - \frac{5\pi}{3}$$

$$V'' = 20\pi - 10\pi r$$

$$V''(4) = 20\pi - 40\pi = -20\pi < 0 \quad \therefore V \text{ is max when } r=4$$

$$\boxed{r=4 \text{ in}}, \boxed{h = 10 - \frac{5\pi}{3} \cdot 4}, \boxed{h = \frac{10}{3} \text{ in}} \quad V_{\max} = \pi \cdot 4^2 \cdot \frac{10}{3} = \boxed{\frac{160\pi}{3} \text{ in}^3}$$

⑤



$$V = 1L \quad SA = 2\pi R^2 + 2\pi RH \quad \text{min}$$

$$\pi R^2 H = 1000 \text{ cm}^3$$

$$H = \frac{1000}{\pi R^2}$$

$$SA = 2\pi R^2 + 2\pi R \cdot \frac{1000}{\pi R^2}$$

$$SA = 2\pi R^2 + \frac{2000}{R}$$

$$(SA)' = 4\pi R - \frac{2000}{R^2} = 0$$

$$4\pi R = \frac{2000}{R^2}$$

$$4\pi R^3 = 2000$$

$$R^3 = \frac{2000}{4\pi}$$

$$R = \sqrt[3]{\frac{2000}{4\pi}} = 5.42 \text{ cm}$$

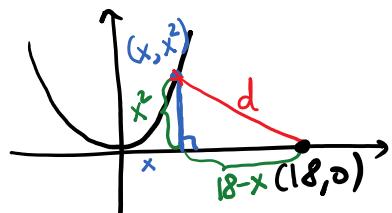
when $R = 5.42$

$$(SA'') > 0$$

$$\boxed{SA \text{ is min when } R = 5.42 \text{ cm}}$$

$$H = 10.84 \text{ cm}$$

⑥



$d \text{ min}$

$$d^2 = (x^2)^2 + (18-x)^2$$

$$d = \sqrt{x^4 + (18-x)^2}$$

$$d' = \frac{1}{2\sqrt{x^4 + (18-x)^2}} \cdot (4x^3 + 2(18-x)(-1))$$

$$d' = \frac{1}{2\sqrt{x^4 + (18-x)^2}} (4x^3 + 2x - 36)$$

$$d' = \frac{1}{\sqrt{x^4 + (18-x)^2}} (2x^3 + x - 18) = 0$$

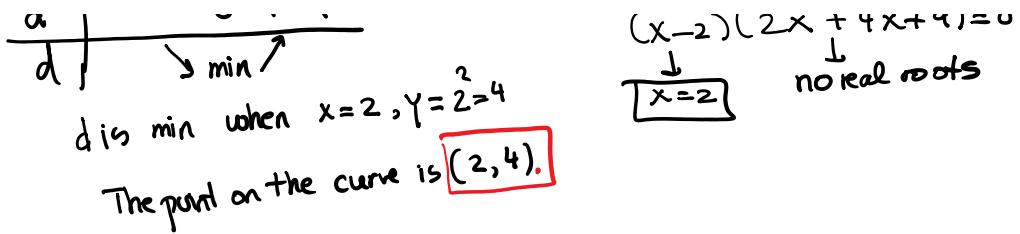
$$\begin{array}{c|cccc} x & & 2 \\ \hline d' & - & 0 & + & + \\ \hline d & \searrow \min \swarrow & & & \end{array}$$

\downarrow $v - 2 \cdot 4 = 2^2 = 4$

$$\begin{array}{rrrrr} 2 & 0 & 1 & -18 \\ 2 & 2 & 4 & 9 & 0 \\ \hline & & & & \end{array}$$

$$(x-2)(2x^2 + 4x + 9) = 0$$

\downarrow $T x = 2$ no real roots



⑦ Profit = Revenue - Cost

$$P = 200x - (500000 + 80x + 0.003x^2)$$

$$P' = 200 - 80 - 0.006x = 0$$

$$-0.006x = -120$$

$$x = 20000 \text{ units}$$

$P'' = -0.006 < 0$
 $\therefore P$ is max when the firm produces $\boxed{20000 \text{ units}}$.