Key

PRECALC 11 MIDTERM REVIEW SHEET

FACTORING (CH.1)

Factor the following expressions:

a) $a^2b^4 - 9a^4$ b) $16x^8 - 1$ c) $m^2 - 5m + 6$ d) $3m^2 - 18m + 24$ e) $6x^2 + 7x - 3$

f) $2(x-2)^2 - 11(x-2) + 15$

QUADRATIC FUNCTIONS (CH. 2)

1. For each parabola: ****
   i) State the coordinates of the vertex
   ii) State the equation of the axis of symmetry
   iii) State the y-intercept and x-intercepts
   iv) State the domain and range
   v) Sketch the graph of each parabola.

a) $y = (x + 2)^2 + 12$ b) $y = -2(x - 3)^2 + 5$ c) $y - 2 = \frac{-1}{2}x^2 - 5$ d) $y = -(x-3)^2$

2. Write each equation in the form $f(x) = a(x - p)^2 + q$. Check by expanding.
   a) $f(x) = x^2 - 8x + 2$ b) $f(x) = -4x^2 - 32x - 50$

The first 3 steps in completing the square below contain one or more errors. Identify the errors, correct them, and then determine the max/min value.

$Y = 2x^2 - 8x + 9$
$Y = 2(x^2 - 8x) + 9$
$Y = 2(x^2 - 8x - 64 + 64) + 9$

4. Sketch each parabola.
   i) Label the vertex with its coordinates.
   ii) Label the axis of symmetry with its equation.
   iii) Label two other points on the graph with their coordinates.

a) $y = x^2 - 24x + 150$ b) $y = -x^2 + 2x + 2$ c) $y = -2(x-3)(x+5)$

5. A quadratic function has zeros −5 and 4. The graph of the function passes through the point (6, 11)
   a) Determine the equation of the function.
   b) Sketch the graph of the function.

6. Determine the quadratic function with a vertex at (5, −4) and passing through the point (2, −1).

7. Determine the equation of a quadratic function that passes through (-2,5), (4,5) and (1,7).

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QUADRATIC EQUATIONS (CH 3)

8. Solve each equation by factoring
a) $x^2 + x - 20 = 0$ b) $2x^2 - 9x - 5 = 0$ c) $(3x + 1)(5x - 4) = 0$

d) $3x^2 = 2x$ e) $3x^2 - 2x = 8$

9. Solve by completing the square: $7 - 2x^2 = -3x$. ****

10. Solve using the quadratic formula: a) $7 - 2x^2 = -3x$ b) $0.25z^2 - 0.1z + 1.36 = 0$

11. Solve by any method: a) $x^4 - 9x^2 = -9 + x^2$ b) $\frac{m}{2} = \frac{m^2-3}{m-1}$

12 ~~8~~. Determine the discriminant and the nature of the roots for each equation. ****
a) $2x^2 + 11x + 5 = 0$ b) $4x^2 - 4x = -1$
c) $2x^2 + 6 = 4x$

13 ~~12~~. For what values of ***k*** does $x^2 + 3x + k = 0$ have two equal real roots?

14 ~~13~~. Determine the values of **k** so $x^2 + kx + (3k - 5) = 0$ will have one real solution

15 ~~14~~. Find 2 numbers whose difference is 16 and whose product is a minimum.

16 ~~15~~. A rectangular horse corral is built using 30 m of fence, where one side of the corral will be an existing barn wall.
a. What is the maximum area the corral can be?
b. What are the dimensions of a corral with an area of 100 m^2?

17 ~~16~~. **Redo various types word problems encountered in Chapters 2, 3 and 4**

SYSTEMS OF EQUATIONS AND INEQUALITIES (CH 4)

18 ~~16~~. Solve the system graphically:
a) $\begin{aligned} 4x - y + 3 &= 0 \\ 2x^2 + 8x - y + 3 &= 0 \end{aligned}$ b) $\begin{aligned} 2x^2 - 16x - y &= -35 \\ 2x^2 - 8x - y &= -11 \end{aligned}$

19 ~~17~~. Solve the system algebraically:
a) $\begin{aligned} y &= -x^2 + 4x - 5 \\ y &= x - 5 \end{aligned}$ b) $\begin{aligned} 5x - y &= 10 \\ x^2 + x - 2y &= 0 \end{aligned}$ c) $\begin{aligned} 3x^2 - x - y - 2 &= 0 \\ 6x^2 + 4x - y &= 4 \end{aligned}$

20 ~~18~~. Graph each inequality
a) $2x - 3y > 12$ b) $-3x + 5y \geq 15$
c) $y < x^2 - 12x + 35$ d) $y \leq x^2 - x - 30$

21 ~~19~~. Solve each inequality
a) $-3x < -15$ b) $0 \leq x^2 - 4x - 5$
c) $-x^2 + 12 \geq -x$ d) $x(x + 6) < 40$

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22 ~~20~~. Graph each region defined by the system of inequalities

a) $x > 1$
$y < -\frac{3}{2}x + 6$
$y > \frac{1}{2}x$

b) $y > 2x - 4$
$y < -\frac{1}{2}x + 1$

23 ~~21~~. Solve the inequalities:
a) $m^2 - 5m + 6 < 0$ b) $a^2 - 4a \leq -4$ c) $4x^2 - 4x + 1 > 0$ d) $-m^2 - 5m + 2 \leq 0$

24 ~~22~~. The sum of two positive real numbers is less than 8 and greater than 4. Show all the possible values of the numbers graphically. How does the answer change if the numbers are positive integers instead?

25 ~~23~~.Rides at the country fair cost \$2 or \$3. Ana can spend at most \$18 on rides. Find all possible combinations of rides that satisfy the problem. Use inequalities!

26 ~~24~~. The length of a rectangle is twice its width. Find possible dimensions of the rectangle if the area is no more than 24 m^2.

a) $a^2(b^2-3a)(b^2+3a)$ b) $(4x^4+1)(2x^2+1)(2x^2-1)$ c) $(m-3)(m-2)$

d) $3(m-4)(m-2)$ e) $(3x-1)(2x+3)$ f) $2a^2-11a+15$

$(2a-5)(a-3)$

$(2x-4-5)(x-2-3)$

$(2x-9)(x-5)$

① a) $V(-2,12)$

axis of sym: $x=-2$

range: $y \geq 12$

y-int: $(0,16)$

x-int: $\emptyset$

b) $V(3,+5)$

$x=3$

$y \leq 15$

$(0,-13)$

$x = \frac{6 \pm \sqrt{10}}{2}$

c) $V(0,-3)$

$x=0$

$y \leq -3$

$(0,-3)$

$x = \emptyset$

d) $V(3,0)$

$x=3$

$y \leq 0$

$(0,-9)$

$x=3$

② $y=(x-4)^2-14$

$y=-4(x+4)^2+14$

③ step 2 ; 4x not 8x

④ a) $V(12,6)$ b) $V(1,3)$ c) $V(-1,32)$

⑤ $y=a(x+5)(x-4)$

$11=a(6+5)(6-4)$

$a=\frac{1}{2}$

$y=\frac{1}{2}(x+5)(x-4)$

$y=\frac{1}{2}x^2+\frac{1}{2}x-10$

⑥ $y=a(x-5)^2-4$

$-1=a(2-5)^2-4$

$a=\frac{1}{3}$

$y=\frac{1}{3}(x-5)^2-4$

⑦ $h=1$

$y=a(x-1)^2+k$

$5=a(4-1)^2+k$

$7=a(1-1)^2+k \Rightarrow k=7$

$a=\frac{-2}{9}$

$\boxed{y=-\frac{2}{9}(x-1)^2+7}$

⑧ a) $-5,4$ b) $5,-\frac{1}{2}$ c) $-\frac{1}{3},\frac{4}{5}$ d) $0,\frac{2}{3}$ e) $2,-\frac{4}{3}$

⑨ $2x^2-3x-7=0$

$2(x^2-\frac{3}{2}x+\frac{9}{16}-\frac{9}{16})-7=0$

$2(x-\frac{3}{4})^2-\frac{9}{8}-7=0$

$2(x-\frac{3}{4})^2=\frac{65}{8}$

$(x-\frac{3}{4})^2=\frac{65}{16}$

$x-\frac{3}{4}=\frac{\pm\sqrt{65}}{4}$

$x=\frac{3\pm\sqrt{65}}{4}$

⑩ $\frac{3\pm\sqrt{65}}{4}$ b) no real roots

⑪ $x^4-10x^2+9=0$ $x=\pm 3$

$(x^2-9)(x^2-1)=0$ $x=\pm 1$

b) $m(m-1)=2(m^2-3)$

$2m^2-6=m^2-m$

$m^2+m-6=0$

$(m+3)(m-2)=0$

$m=-3, m=2$

⑫ a) 81, 2 different real roots

b) 0; 2 equal roots

c) -32; no real roots

⑬ $k=2.25$

⑭ 10, 2

⑮ 8, -8

⑯ a) $112.5\,m^2$ b) 5m by 20m

(18) use Desmos to check your answers

(19) a) $-x^2+4x-5 = x-5$

$x^2-3x=0$

$x=0$, $x=3$

$y=-5$, $y=-2$

$(0,-5)$, $(3,-2)$

b) $(4,10)$, $(5,15)$

c) $(-2,12)$
$(\frac{1}{2},-2)$

(20) check with Desmos

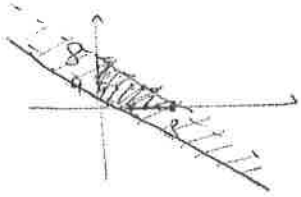
(21) a) $x>5$ b) $x>5$ or $x<-1$ c) $-3\le x\le 4$ d) $-10\le x\le 4$

(23) a) $2<m<3$ b) $a=2$ c) all real # d) $m^2+5m-2\ge 0$

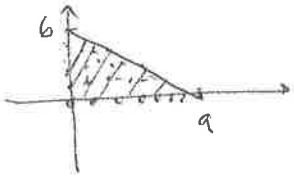
$\frac{-5-\sqrt{33}}{2}$, $\frac{-5+\sqrt{33}}{2}$

$m<\frac{-5-\sqrt{33}}{2}$ or $m>\frac{-5+\sqrt{33}}{2}$

(24) $\begin{cases} x+y<8 \Rightarrow y<-x+8 \\ x+y\ge 4 \Rightarrow y\ge -x+4 \end{cases}$



(25) $\begin{cases} 2x+3y\le 18 \Rightarrow y\le -\frac{2}{3}x+6 \\ x\ge 0 \\ y\ge 0 \end{cases}$



(26) $\begin{cases} L=2w \\ Lw\le 24 \\ w>0 \\ L>0 \end{cases}$ $\Rightarrow 2w^2\le 24$

$w^2\le 12$

$-\sqrt{12}\le w\le\sqrt{12}$

$$\boxed{\begin{array}{l} 0<w\le\sqrt{12} \\ 0<L\le 2\sqrt{12} \end{array}}$$