Review for the Midterm

) $\lim_{x \to 6} \frac{\sqrt{x-2}-2}{\sqrt{x-6}} =$ 0

b) $\lim_{x \to \infty} \left(1+\frac{a}{x}\right)^{bx} =$ e^{ab}

c) $\lim_{t \to \infty} \frac{3t^2+8t-6}{t^2-1} =$ 3

d) $\lim_{x \to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3} =$ $\frac{-1}{9}$

e) $\lim_{x \to 0} \frac{\sin 4x}{-5x} =$ $-\frac{4}{5}$

f) $\lim_{x \to 1^-} \frac{|x-1|}{x^2-1} =$ $-\frac{1}{2}$

2. Find the equations of the asymptotes for $y = \frac{-3x^2}{x^2-4}$.

$x = \pm 2$, $y = -3$ Prove using limits

2. Differentiate: a) $f(x) = e^x - \frac{1}{x^2} + \sqrt[5]{x^2} - 10^3$

$f'(x) = e^x + 2x^{-3} + \frac{2}{5}x^{-\frac{3}{5}}$

b) $F(v) = \left(\frac{v}{v^3+1}\right)^6$

$F'(v) = \frac{6v^5(1-2v^3)}{(v^3+1)^7}$

c) $k(x) = \sin x \cos(x^2)$

$k'(x) = \cos x \cdot \cos x^2 - 2x \sin x \cdot \sin x^2$

d) $g(x) = \frac{\tan x}{x - \sec x}$

hint $\times \frac{\cos x}{\cos x}$

d) $g'(x) = \frac{x - \cos x - \sin x \cos x}{(x\cos x - 1)^2}$

3. a) State the definition of the derivative of a function $f(x)$.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

b) Use the **definition** to find $f'(x)$, given that $f(x) = \sqrt{9-x}$.

$\lim_{h \to 0} \frac{\sqrt{9-x-h} - \sqrt{9-x}}{h} = \lim_{h \to 0} \frac{(9-x-h)-(9-x)}{h(\sqrt{9-x-h}+\sqrt{9-x})} = \lim_{h \to 0} \frac{-h}{h(\sqrt{9-x-h}+\sqrt{9-x})} = \frac{-1}{2\sqrt{9-x}}$

4. a) State the definition of continuity of a function f at a number a.

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

) For what values of the constant c is the function f continuous on $(-\infty, \infty)$? Justify your answer using the definition of ntinuity.

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

$\lim_{x \to 2^-} (cx^2 + 2x) = 4c + 4$

$\lim_{x \to 2^+} (x^3 - cx) = 8 - 2c$

$f(2) = 8 - 2c$

$8 - 2c = 4c + 4$

$c = \frac{4}{6} = \boxed{\frac{2}{3}}$

Review for the Midterm

5. Suppose the position of an object moving horizontally after t seconds is given by

$$s = f(t) = 2t^3 - 21t^2 + 60t, \qquad 0 \le t \le 6,$$

where s is measured in metres, with $s > 0$ corresponding to positions right of the origin.

a) Find the velocity function. When is the object stationary, moving to the right, and moving to the left?

$v(t) = 6t^2 - 42t + 60$ + 2 − 5 +

object is stationary when $v(t) = 0$, $t = 2$ or $t = 5$

moving right $0 < t < 2$ and $5 < t < 6$

left $2 < t < 5$

b) Determine the velocity and acceleration of the object at $t = 1$.

$v(1) = 24$ m/sec

$a(t) = 12t - 42$ $a(1) = -30$ m/sec^2

c) Determine the acceleration of the object when its velocity is zero.

$v = 0 \Rightarrow t = 2$ or $t = 5$

$a(2) = -18$ $a(5) = 18$

d) On what intervals is the object speeding up? On what intervals is it slowing down?

when $v > 0, a > 0$ or $v < 0$, $a < 0$ (same sign)

	v	a	
$0<t<2$	+	−	slowing down
$2<t<\frac{7}{2}$	−	−	speeding
$\frac{7}{2}<t<5$	−	+	slowing
$5<t<6$	+	+	speeding

$v > 0, a < 0$ or $v < 0, a > 0$ (opposite signs)

6. Determine an equation of the tangent line to the curve $x^4 - x^2y + y^4 = 1$ at the point (-1, 1).

$4x^3 - 2xy - x^2y' + 4y^3 y' = 0$

$y' = \dfrac{2xy - 4x^3}{4y^3 - x^2}$ slope $= \frac{2}{3}$

tangent line: $y - 1 = \frac{2}{3}(x + 1)$

7. Find a curve with the following properties:

a) $\frac{d^2y}{dx^2} = 6x$ $y = x^3 + Cx + D$ $y' = 3x^2 + C$

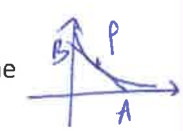
b) Its graph passes through the point (0,1) and has a horizontal tangent at this point.

(0,1) $D = 1$

(0,1) $0 = C$

$y = x^3 + 1$

8. Consider the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ in the first quadrant. Show that the length of segment XY of a tangent line to the curve at a point P cut off by the coordinate axes is constant and find this length.



Let $P(x_0, y_0)$

$y' = -\left(\frac{y}{x}\right)^{1/3}$ eq. of the tangent line: $y - y_0 = -\left(\frac{y_0}{x_0}\right)^{1/3}(x - x_0)$

y-intercept: $4y_0^{1/3}$

y-int: $4x_0^{1/3}$

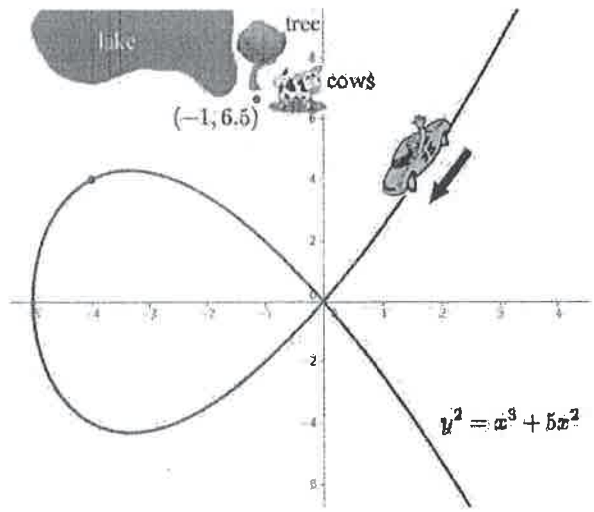
AB length $= \sqrt{(4x_0^{1/3})^2 + (4x_0^{1/3})^2} = 8$

9. Given $x^{\cos y} = y^{\cos x}$, find $\frac{dy}{dx}$ in terms of x and y.

$\ln x^{\cos y} = \ln y^{\cos x} \Rightarrow \cos y \ln x = \cos x \ln y$ then differentiate

$$\frac{dy}{dx} = \frac{\left(\frac{\cos y}{x} - \cos x \ln y\right)}{\frac{\sin x}{y} + \sin y \ln x}$$

10. A race car is speeding around a race-track and comes to a particularly dangerous curve in the shape $y^2 = x^3 + 5x^2$. The diagram below indicates the direction the car is traveling along the curve.



[2] (a) Find the derivative of y with respect to x.

$$2y\frac{dy}{dx} = 3x^2 + 10x$$

$$\frac{dy}{dx} = \frac{3x^2+10x}{2y}$$

[3] (b) If the car skids off at the point $(-4, 4)$ and continues in a straight path find the equation of the line the car will travel in.

$$\left.\frac{dy}{dx}\right|_{\substack{x=-4 \\ y=4}} = 1$$

$$\boxed{y = x+8}$$

[1] (c) If a tree is located at the point $(-1, 6.5)$ with a lake to the left and cows to the right, will the car hit the lake, the tree or the cows?

since point (−1, 7) lies on the tangent line the car would hit the lake