Name: _____ Blk: ____ Date: _____

PreCalculus 12

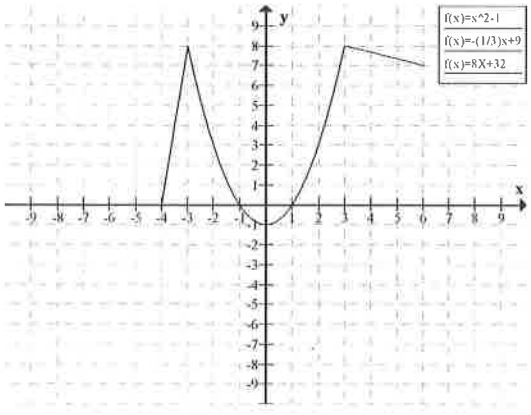
Final Exam Review

CH 1.1-1.3: OPERATIONS WITH FUNCTIONS

1) If $f(x) = (3x-1)^2$ and $g(x) = (x+2)^2$, determine $f(x)+g(x), f(x)-g(x) and\ f(x) \div g(x)$.
2) If $f(x) = 3x+2\ and\ g(x) = 2x-5$, what is $g(f(3))$?
3) If $f(x) = x^2 - 4x$ and $g(x) = 3x-1$, what is $f(g(x))$?
4) If $g(x) = 2-5x$, what is $g(g\left(-\frac{2}{3}\right))$?
5) If $f(x) = \sqrt{6x}$ and $g(x) = 2x+3$ what is $f(g(x))$?
6) If $f(x) = x-2$ and $g(x) = \sqrt{x+1}$, what is the domain of $f(g(x))$?
7) The area of a circle, A, in terms of its radius, r, is given by $A(r) = \pi r^2$. The radius in terms of its circumference, C if $r(C) = \frac{C}{2\pi}$. What is the function that expresses the area of the circle in terms of its circumference?

CH 1.4-1.6: TRANSFORMATIONS OF FUNCTIONS

1) Which equation represents the graph of $y = 2^x$ after it is reflected in the x-axis.
   a) $y = 2^{-x}$ b) $y = -2^x$ c) $y = \log_2 x$ d) $y = -\log_2 x$
2) How is the graph of $y = f(4x)$related to the graph of $y = f(x)$?
   a) $y = f(x)$ has been compressed vertically by a factor of $\frac{1}{4}$.
   b) $y = f(x)$ has been compressed horizontally by a factor of $\frac{1}{4}$.
   c) $y = f(x)$ has been expanded vertically by a factor of 4.
   d) $y = f(x)$has been expanded horizontally by a factor of 4.
3) If the maximum value of the function $y = f(x)$ is 6, determine the maximum value of $y = \frac{1}{3}f\left(\frac{1}{2}x\right)$.
4) If the point (-2,-5) is on the graph of $y = f(x)$, which point must be on the graph of $y = f(1-x)+3$
5) The graph of $y = f(x)$ is shown below.



   a) On the grid provided, sketch the graph of $y = 0.5\,f(-x+1)-2$.
   b) On the grid provided, sketch the graph of $y = -f(2x-2)+1$
6) If the graph of $2x+3y=5$ is translated 4 units up, determine an equation of the new graph.
7) If the point $(2,-8)$ is on the graph of $y = f(x-3)+4$, what point must be on the graph of $y = f(x)$?
8) Given $f(x) = \frac{x}{3x-1}$, determine $f^{-1}(x)$, the inverse of $f(x)$.

9) Given $f(x) = \frac{2x}{1-x}$, determine $f^{-1}(x)$, the inverse of $f(x)$.

10) The zeroes of a function $y = f(x)$ are -3, 0, 2. Determine the zeroes of the function $y = f(2 - x)$.

11) If $(3, -4)$ is a point on the graph of $y = f(x)$, what must be a point on the graph of

$y = \frac{1}{2} f(1 - x) - 2$?

12) If (m, n) is a point on the graph of $y = f(x)$, determine a point on the graph $y = -f(4x - 2) + 1$.

13) If the graph $x^2 + y^2 = 1$ is horizontally expanded by a factor of 3 and vertically compressed by a factor of $\frac{1}{2}$, determine an equation for the new graph.

CH 3: RADICAL FUNCTIONS AND EQUATIONS

1) Solve by algebra and graphing:

a) $\sqrt{x-2} = 2$ b) $\sqrt{5x-6} = x$ c) $\sqrt{13-x} - x + 1 = 0$ d) $\sqrt[3]{x+6} = 2$

2) Determine the domain, range, the x-intercept, the y-intercept and graph the function $y = -2\sqrt{4-2x} + 3$.

3) Determine the domain, range, the x-intercept, the y-intercept and graph the function $y = -2\sqrt{x^2-4} + 1$.

CH 2: POLYNOMIAL FUNCTIONS

1) What is the greatest number of roots that $x^4 + 9x^3 - 3 = 2x^5 - 11x^2$ could have?

2) Which cubic function has zeros of $-3, -3, 2$?

a) $y = -8(x-3)^2(x+2)$ b) $y = 2(x-3)^2(x+2)^2$ c) $y = 4(x+3)(x-2)$

d) $y = -5(x+3)(x-2)^2$ e) $y = (x+3)^2(x-2)$

3) Which quartic function has zeros $-5, -2, 2, 3$?

a) $y = 4(x+5)(x+2)(x-2)(x-3)$
b) $y = -2(x+5)(x-2)(x-3)$
c) $y = -7(x-5)(x-2)(x-2)(x-3)$
d) $y = 3(x-5)(x-2)(x+2)(x+3)$
e) $y = x^4 - 2x^3 + 5x^2 - 12x + 16$

4) Which of the following statements is false?

a) A quartic function could have two pairs of equal real zeros.
b) A cubic function could have just one distinct real zero
c) A quintic function must have at least one real zero
d) A quadratic function could have a double zero
e) A polynomial function must have at least one real zero

5) Graph: $y = 2x(x-3)^2(x+2)^3$.
6) Solve $8x^3 + 18x^2 - 56x = 0$
7) What is the remainder when $x^3 - 3x^2 + 7x + 5$ is divided by $x + 4$?
8) What is the value of k if $x^3 + kx^2 + 7x + 12$ is divided by $x + 2$, and gives a remainder of 2?
9) What is the value of k if $x^4 + kx^3 + x^2 - 6x + 3$ is divided by $x + 3$, and gives a remainder of -24?
10) When the polynomial $3x^2 - bx + 20$ is divided by $x - 4$, the remainder is -12. What is the remainder when the polynomial is divided by $x + 2$?
11) The polynomial $ax^3 + bx^2 - 4x + 7$ when divided by $x + 5$ has a remainder of 2 and when divided by $x + 1$ has a remainder of 30. What is the value of ?
12) What is the value of $f(2)$ if $x - 2$ is a factor of $f(x)$?
13) Which values of x should be chosen to test for factors of $2x^4 + 7x^3 - 4x^2 + 2x - 12$?
14) Without dividing, what is the remainder for $(6x^2 + x - 4) \div (3x - 4)$?
15) Solve by factoring. $20x^3 + 37x^2 - 63x + 18 = 0$
16) Solve by factoring. $4x^3 + 14x^2 + 8x + 1 = 0$
17) Without dividing, what is the remainder for $(8x^2 + 2x - 5) \div (2x - 1)$?
18) Solve the inequality $(a - 2)(a + 4) > 0$
19) Solve the inequality $x(x - 3) > 0$
20) Which graph represents the solution for $x^2 - 4x - 12 \geq 0$?
21) Solve the inequality: $(x + 4)(x - 2)(x - 6) > 0$
22) Solve the inequality: $x^3 - 4x^2 - 11x + 30 \leq 0$

CH 4: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1) $27^x(9^{2x-1}) = 3^{x+4}$
2) The half-life of sodium-24 is 14.9 h. A hospital buys a 40-mg sample of sodium-24.
   a) How many grams to the nearest tenth, of sodium-24 will remain after 48h?
   b) After how long will only 2.5mg remain?
3) A bacterium is quadrupling every seven days.
   a) Write an exponential function the models the growth of the number of bacteria.
   b) How many times as great will the number of bacteria be in three weeks as the number now?
   c) How long will it take the number of bacteria to double?
   d) How long ago was there only 25% of the current number of bacteria?
   e) After how long will a single bacterium grow to 8^{24} bacteria?
4) Evaluate without a calculator: $\log_{\sqrt{7}} 7^3$.
5) As an iceberg melts during the summer, it loses 3% of its mass every 5 days. This iceberg reduces to 40% of its original mass after t days. Write an equation which could be used to determine the value of t?
6) Solve: $\log_2(\log_9 x) = -1$
7) Solve: $5^{x+1} = 2(3^{2x})$
8) Change to logarithmic form $a^3 = b$.
9) Give the domain of $f(x) = \log_7(-2x + 6) + 12$
10) Express $\log_5 30$ using logarithms in base 4.
11) Expand: $log \frac{x}{2y^3}$.
12) Solve: $\left(\frac{1}{9}\right)^x = 27^{2-x}$
13) Solve: $\log_2 x + \log_2(x - 1) = 3$

14) Determine an exponential function in the form $y = 3^{x-h}+k$ with a y-intercept 5 and asymptote $y = -4$.

15) The population of a nest of ants can multiply threefold (triple) in 8 weeks. If the population is now 12000, how many weeks will it take for the population to reach 300,000 ants?
(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

16) Express as a single logarithm: $\log m - \log n - 3\log k$

17) Determine the domain of the function $y = \log_{x-1}(5-x)$.

18) Simplify: $9\log_{27} x - 4\log_9 x$

a) $\log_3 x$ b) $\log_9 x$ c) $\log_{27} x$ d) $\frac{3}{4}\log_3 x$

19) A particular type of bacteria multiplies 5-fold every 30 minutes. Initially there are 100 bacteria. Determine an expression for the number of bacteria after *k* minutes.

20) Given $f(x) = 3(2^{x-2}) + 5$, determine $f^{-1}(x)$, the inverse of $f(x)$.

21) Solve algebraically: $2\log_3(x+4) - \log_3(-x) = 2$

22) Change $\log_{2a} p = t$ to exponential form.

23) Determine an equivalent expression for $\log a - 2\log b - 3\log c$.

24) Solve: $\log_5(3x) - \log_5(x-3) = 2$

25) Solve: $9^{x+2} = (3^{4x-3})(3^5)$

26) In chemistry, the pH-scale measures the acidity (0-7) or alkalinity (7-14) of a solution. It is a logarithmic scale in base 10. Thus a pH of 5 is 10 times more acidic than a pH of 6. Solution A has a pH of 5.7. Solution B is 1260 times more acidic than Solution A. Find the pH of solution B.

27) A radioactive substance has a half-life of 17 d. How long will it take for 300 g of this substance to decay to 95 g? (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

28) Solve for x: $ab^x = c$

29) Solve algebraically using logarithms: $2^x = 3(5^{x+1})$ (Answer accurate to at least 2 decimal places)

30) Solve for x: $\log(3-x) + \log(3+x) = \log 5$

31) Solve: $\log_2 8 + \log_3 \frac{1}{3} = \log_4 x$

32) Solve the following: $\log_2(\log_4(\log_5 x)) = -1$

33) Solve algebraically: $2\log_4 x - \log_4(x+3) = 1$

34) Write as a single logarithm: $3 + \frac{1}{2}\log_2 x - 3\log_2 y$

35) If $\log_4 x = a$, determine $\log_{16} x$ in terms of a.

36) If $\log 2 = a$, $\log 3 = b$, determine an expression for $\log 2400$

37) Simplify: $a^{\log_a 8 + \log_a 2}$

38) Determine the value of $\log_n ab^2$ if $\log_n a = 5$ and $\log_n b = 3$.

CH 5 – 6: TRIGONOMETRY

1) Determine the general solution algebraically. (Solve over the set of real numbers)

$3\cos^2 x - 8\cos x + 4 = 0$ (Answer accurate to at least 2 decimal places.)

2) Prove the identity : $\frac{\tan x(\cos x + \cot x)}{\sec x + \tan x} = \frac{\sin x \sin 2x}{2 - 2\cos^2 x}$

3) A circle has a radius of 20cm. Determine the length of the arc subtended by a central angle of 135°.

4) Determine the exact value: a) $\sec\frac{4\pi}{3}$ b) $\tan\frac{7\pi}{6}$ c) $\sin(-\frac{3\pi}{4})$.

5) Solve: a) $\csc x = 2, 0 \le x < 2\pi$ b) $\sin 2x = \frac{1}{\sqrt{2}}, where\ 0 \le x < 2\pi$.

6) Solve algebraically, giving exact values, where $0 \leq x < 2\pi$. $\sin x = \cos 2x$.
7) Solve algebraically, giving exact values, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$: $2\tan x \cos x - \sqrt{3}\tan x = 0$.
8) Solve algebraically, giving exact values: $\sin\frac{1}{3}x = \frac{\sqrt{3}}{2}$
   a) Where $0 \leq x < 2\pi$
   b) Over the set of real numbers.
9) The two smallest positive solutions of $\sin 3x = 0.7$ *are* $x = 0.26$ *and* $x = 0.79$. Determine the general solution for $\sin 3x = 0.7$.
10) Solve algebraically $6sin^2x = \sin x + 2$ over the set of real numbers. (Give exact value solutions where possible, otherwise answer accurate to two decimal places.)
11) Solve algebraically $\sin 2x - 2cos^2x = 0$ over the set of real numbers. (Give exact value solutions)
12) Determine the restriction(s) for the expression $\frac{\tan\theta}{2\cos\theta - 1}$
13) Determine an expression equivalent to $tan^2\theta \csc\theta + \frac{1}{\sin\theta}$.
14) Simplify: $3\cos 2x \cos x + 3\sin 2x \sin x$.
15) Simplify: $\frac{6\sin\theta}{\sin 2\theta}$.
16) Prove the identity: $\frac{\tan x + \sin x}{1+\cos x} = \frac{1}{\csc 2x} - \frac{\tan x}{\sec 2x}$.
17) The terminal arm of angle θ in standard position passes through the point (-2, 5). Determine the value of $\sec\theta$.
18) Determine the amplitude, period and the max and min values of $y = -3\cos(2x - \frac{\pi}{3}) + 2$. Then, graph the function and label five key points in one period.

CH 7: COMBINATORICS

1) When you play lotto 5-30, you must choose 5 different integers from 1 to 30. How many combinations are possible?
2) Determine the 4th term of $(3x - 2)^6$
3) Determine the number of different arrangements of all the letters in APPLEPIE.
4) Assume a car license plate consists of 7 characters. The first 3 characters can be any of the letters from A to F, but no letter can be repeated. The next 3 characters can be any of the digits from 1 to 9, but no digit can be repeated. The last character can be any of the letters X,Y or Z. An example of this format is: BFA648Y. How many license plates are possible?
5) Suppose you play a game of cards in which only three cards are dealt from a standard 52-card deck. How many ways are there to obtain one pair? (2 cards of the same rank and 1 card of a different rank.) An example of a hand that contains one pair is 2 jacks and 1 five.
6) A soccer coach must choose 3 out of 10 players to kick tie-breaking penalty shots. Assuming the coach must designate the order of the 3 players, determine the number of different arrangements she has available.
7) Determine the 4th term in the expansion of $(x - 2y)^5$.
8) Solve algebraically : $\frac{(n-1)!}{(n-3)!} = 30$
9) Express ${}_{33}C_5$ using factorial notation.
10) Determine the middle term in the expansion of $(x - y)^{10}$.
11) A class has 30 students.
    a) How many ways can a committee of 3 people be selected from the class?
    b) How many ways can an executive committee consisting of 3 people (president, vice-president, secretary) be selected from the class?

c) If there are 10 boys and 20 girls in the class, how many ways can a committee of 3 people be selected from the class if the committee must contain 1 boy and 2 girls.

12) How many different pasta meals can be made from 4 choices of pasta and 2 choices of sauces, if only one pasta and one sauce is selected for each meal?

13) A man has 7 different pets and wishes to photograph them 3 at a time arranged in a line. How many different arrangements are possible?

14) Suppose you play a game of cards in which only four cards are dealt from a standard deck of 52 cards. How many ways are there to obtain three of a kind? (3 cards of the same rank and 1 card of a different rank, for example 3 tens and 1 queen.)

15) How many permutations are there using all of the letters in the word P E P P E R?

16) In a particular city, all of the streets run continuously north-south or east-west. The mayor lives 4 blocks east and 5 blocks north of city hall. Determine the number of different routes, 9 blocks in length that the mayor can take to get to city hall.

17) In the expansion of $(x + y)^{10}$, determine the coefficient of the term containing x^8y^2.

18) In a standard deck of 52 cards, how many different 4-card hands is there that contain at most one heart?

19) In a library, 4 different English books, 2 different Chemistry books and 3 different Mathematics books are arranged on a shelf. Determine the number of different arrangements if the books on each subject must be kept together.

Operations with Functions

① $f(x)+g(x)=(3x-1)^2+(x+2)^2=10x^2-2x+5$

$f(x)-g(x)=8x^2-10x-3$

$f(x)\div g(x)=\dfrac{(3x-1)^2}{(x+2)^2}$, $x\neq -2$

② $g(f(3))=g(11)=2\cdot 11-5=22-5=17$

③ $f(g(x))=(3x-1)^2-4(3x-1)=9x^2-18x+5$

④ $g(g(-\frac{2}{3}))=g(2+\frac{10}{3})=g(\frac{16}{3})=2-5\cdot\frac{16}{3}=\frac{6-80}{3}=\frac{-74}{3}$

⑤ $f(g(x))=\sqrt{6(2x+3)}=\sqrt{12x+18}$

⑥ $f(g(x))=\sqrt{x+1}-2$; $x\geq -1$

⑦ $A(r)=\pi r^2$ $\quad A(C)=\pi\left(\frac{C}{2\pi}\right)^2=\pi\cdot\frac{C^2}{4\pi^2}=\frac{C^2}{4\pi}$

$r(C)=\frac{C}{2\pi}$

CH. 1.4-1.6 Transformations of Functions

① $y\mapsto -y$ (b)

② (b)

③ $(a,6)\xrightarrow{\times\frac{1}{3}}$ 2

④ $y=f(-(x+1))+3$

$(-2,5)\to(2,-5)\to(3,-5)\to(3,-2)$

⑤ a) $y=\frac{1}{2}f[-(x-1)]-2$ $\quad$ b) $y=-f[2(x-1)]+1$

(6) $y \mapsto y-4$ $\quad 2x+3(y-4)=5$

(7) ~~$(2,-8)$~~ $\to$ ~~$(5,-8)$~~ $\to$ ~~$(5,-4)$~~ $(-1,-12)$

(8) $x = \dfrac{y}{3y-1}$

$x(3y-1)=y$

$3xy-x=y$

$3xy-y=x$

$y(3x-1)=x$

$y=\dfrac{x}{3x-1}$

$f^{-1}(x)=\dfrac{x}{3x-1}$

(9) $y=\dfrac{2x}{1-x}$

$x=\dfrac{2y}{1-y}$

$x-xy=2y$

$x=xy+2y$

$x=y(x+2)$

$y=\dfrac{x}{x+2}$

$f^{-1}(x)=\dfrac{x}{x+2}$

(10) $y=f(2-x)=f[-(x-2)]$

$(-3,0) \to (3,0) \to (5,0)$

$(0,0) \to (0,0) \to (2,0)$

$(2,0) \to (-2,0) \to (0,0)$

(11) $y=\frac{1}{2}f(-(x+1))-2$

$(3,-4) \to (3,-2) \to (-3,-2) \to (-2,-2) \to (-2,-4)$

(12) $(m,n) \to (m,-n) \to \left(\frac{m}{4},-n\right) \to \left(\frac{m}{4}+\frac{1}{2},-n\right) \to \left(\frac{2+m}{4},-n+1\right)$

(13) $x \to \frac{1}{3}x$

$y \to 2y$

$\left(\frac{x}{3}\right)^2+(2y)^2=1$

Ch.3 Radical Functions and Equations

① a) $\sqrt{x-2} = 2 \quad ; x \geq 2$

$x-2 = 4$

$x = 6$

Check: $\sqrt{6-2} = 2$

$2 = 2$ ✓

$S = \{6\}$

b) $\sqrt{5x-6} = x$

$5x-6 = x^2$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2$ or $x = 3$

Check: $\sqrt{5 \cdot 2 - 6} = 2$

$2 = 2$ ✓

$\sqrt{15-6} = 3$

$3 = 3$ ✓

$S = \{2, 3\}$

c) $\sqrt{13-x} = x-1$

$13 - x = x^2 - 2x + 1$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ or $x = -3$

Check: $\sqrt{13-4} = 4-1$

$3 = 3$ ✓

$\sqrt{13+3} = -3-1$ ✗

$S = \{4\}$

d) $\sqrt[3]{x+6} = 2$

$x+6 = 8$

$x = 2$

Check ✓

$S = \{2\}$

② $y = -2\sqrt{4-2x} + 3$

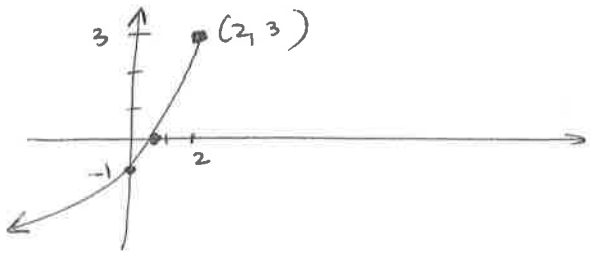
$4 - 2x \geq 0$

$4 \geq 2x$ ✓

$x \leq 2$

$d = \{x \mid x \leq 2, x \in \mathbb{R}\}$

$r = \{y \mid y \leq 3, y \in \mathbb{R}\}$



$x = 0 \quad y = -2 \cdot 2 + 3 = -1 \quad (0, -1)$

$y = 0 \quad -3 = -2\sqrt{4-2x}$

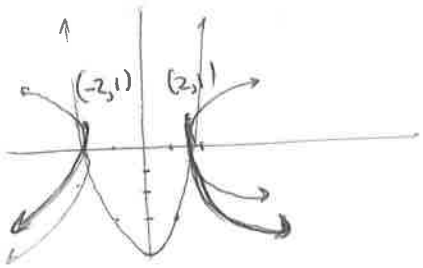
$\left(\frac{3}{2}\right)^2 = 4 - 2x$

$4 - \frac{9}{4} = 2x \quad \left(\frac{7}{8}, 0\right)$

$2x = \frac{7}{4}$

$x = \frac{7}{8}$

③ $y = -2\sqrt{x^2-4} + 1$



$x^2 - 4 \geq 0 \Rightarrow \quad x \leq -2$ or $x \geq 2$

$d = \{x \mid x \leq -2 \text{ or } x \geq 2, x \in \mathbb{R}\}$

$r = \{y \mid y \leq 1\}$

$x = 0$ no y-intercept

$y = 0 \quad -1 = -2\sqrt{x^2-4}$

$\frac{1}{4} = x^2 - 4$

$x^2 = \frac{17}{4}$

$x = \pm \frac{\sqrt{17}}{2}$

Ch. 2 Polynomial Functions

(1) 5

(2) e

(3) a

(4) e

(5) $y = -\frac{1}{2}(x-6)(x-2)(x+3)$

(6) $-4, 0, \frac{7}{4}$

(7) -135

(8) 3

(9) ~~4~~ 5

(10) 72

(11) 24

(12) 0

(13) $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$, $\pm \frac{1}{2}, \pm \frac{3}{2}$

(14) 8

(15) $-3, \frac{2}{5}, \frac{3}{4}$

(16) $-\frac{1}{2}, \frac{-3 \pm \sqrt{7}}{2}$

(17) -2

(18) $a < -4$ or $a > 2$

(19) $x < 0$ or $x > 3$

(20)

(21) $-4 < x < 2$ or $x > 6$

(22) $x \leq -3$ or $2 \leq x \leq 5$

CH. 4

(1) $3^{3x} \cdot 3^{2(2x-1)} = 3^{x+4}$

$3x + 4x - 2 = x + 4$

$6x = 6$

$\boxed{x = 1}$

(2) $A = A_0 x^{\frac{t}{1}}$

a) $A = 40\left(\frac{1}{2}\right)^{\frac{48}{14.9}} = 4.3g$

b) $2.5 = 40\left(\frac{1}{2}\right)^{\frac{t}{14.9}}$

$t = 59.6h$

(3) a) $A(t) = A_0(4)^{\frac{t}{7}}$

b) 64 times

c) 3,5 days

d) 7 days ago

e) 252 days

(4) 6

(5) $40 = 100(0.97)^{\frac{t}{5}}$

(6) $x = 9^{\frac{1}{2}} = 3$

(7) $(x+1)\log 5 = \log 2 + 2x \log 3$

$x = \frac{\log 2 - \log 5}{\log 5 - 2\log 3}$

(8) $\log_a b = 3$

(9) $-2x + 6 > 0$

$\boxed{x < 3}$

(10) $\frac{\log_4 30}{\log_4 5}$

(11) $\log x - \log 2 - 3\log y$

(12) $3^{-2x} = 3^{3(2-x)}$

$-2x = 6 - 3x$

$\boxed{x = 6}$

(13) $x(x-1) = 8$ $\quad x>0$, $x>1$

$x^2 - x - 8 = 0$

$x = \frac{1 \pm \sqrt{33}}{2}$

$\boxed{x = \frac{1+\sqrt{33}}{2}}$

(14) $y = 3^{x-h} - 4$

$x = 0 \quad 5 = 3^{-h} - 4$

$9 = 3^{-h}$

$h = -2$

$\boxed{y = 3^{x+2} - 4}$

(15) $A = 12000(3)^{\frac{t}{8}}$

$\frac{300000}{12000} = \frac{12000}{12000} \cdot 3^{\frac{t}{8}}$

$t = 23.4$

(16) $\log \frac{m}{nk^3}$

(17) $\begin{cases} 5 - x > 0 \\ x - 1 > 0 \\ x - 1 \neq 1 \end{cases} \rightarrow \boxed{1 < x < 5,\ x \neq 2}$

(18) a

(19) $A = 100(5)^{\frac{k}{30}}$

(20) $x = 3 \cdot 2^{y-2} + 5$

$\frac{x-5}{3} = 2^{y-2}$

$y - 2 = \log_2\left(\frac{x-5}{3}\right)$

$y = f^{-1}(x) = \log_2\left(\frac{x-5}{3}\right) + 2$

(21) $\frac{(x+4)^2}{-x} = 9 \quad \begin{cases} x+4 > 0 \\ -x > 0 \end{cases}$

$x^2 + 8x + 16 + 9x = 0$

$x^2 + 17x + 16 = 0$

$(x+16)(x+1) = 0$

$x = -16, \boxed{x = -1}$

↓ reject

(22) $(2a)^t = p$

(23) $\log \frac{a}{b^2 c^3}$

(24) $\frac{3x}{x-3} = 25$

$3x = 25x - 75$

$x = \frac{75}{23}$

(25) $2x + 4 = 4x - 3 + 5$

$x = \frac{2}{2} = 1$

(26) 2.6

(27) 28.20 days

(28) $b^x = \frac{c}{a}$

$x = \log_b \frac{c}{a}$

(29) $x\log 2 = \log 3 + (x+1)\log 5$

$x = \frac{\log 3 + \log 5}{\log 2 - \log 5}$

(30) $9 - x^2 = 5$ $\begin{cases} 3-x>0 \\ 3+x>0 \end{cases}$

$x^2 = 4$

$\boxed{x = 2 \text{ or } -2}$

(31) $3 + (-1) = \log_4 x$

$2 = \log_4 x$

$x = 4^2 = 16$

(32) $\log_4(\log_5 x) = \frac{1}{2}$

$\log_5 x = 4^{\frac{1}{2}} = \sqrt{4} = 2$

$x = 25$

(33) $x = 6$

(34) $\log_2 \frac{8\sqrt{x}}{y^3}$

(35) $\log_{16} x = \frac{\log_4 x}{\log_4 16} = \frac{a}{2}$

(36) $\log 2400 = \log(8 \cdot 3 \cdot 100)$

$= \log 8 + \log 3 + \log 100$

$= 3\log 2 + \log 3 + 2$

$= 3a + b + 2$

(37) $a^{\log_a 16} = 16$

(38)

$\log_n a + 2\log_n b =$

$= 5 + 2 \cdot 3 = 11$

CH. 5-6 TRIGONOMETRY

① $(3\cos x - 2)(\cos x - 2) = 0$

$\cos x = \frac{2}{3}$ $\quad \cos x = 2$ no solution

$x_1 = \cos^{-1}\left(\frac{2}{3}\right) = 0.8411$

$x_2 = 2\pi - \cos^{-1}\left(\frac{2}{3}\right) = 5.4421$

$0.8411 + 2n\pi$
$5.4421 + 2n\pi$, n-integer

③ $l = R\theta_{rad}$

$l = 20 \cdot \frac{3\pi}{4} = 15\pi$ cm

④ a) $\sec\frac{4\pi}{3} = -2$ b) $\tan\frac{7\pi}{6} = -\frac{\sqrt{3}}{3}$ c) $\sin\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

⑤ a) $\sin x = \frac{1}{2}$

$x_1 = \frac{\pi}{6}$

$x_2 = \frac{5\pi}{6}$

b) $\sin 2x = \frac{1}{\sqrt{2}}$

$\sin a = \frac{1}{\sqrt{2}}$

$a_1 = \frac{\pi}{4}$

$a_2 = \frac{3\pi}{4}$

$x_1 = \frac{\pi}{8}$, $x_2 = \frac{3\pi}{8}$

$x_3 = \frac{9\pi}{8}$, $x_4 = \frac{11\pi}{8}$

⑥ $\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = \frac{3\pi}{2}$

⑦ $\tan x\,(2\cos x - \sqrt{3}) = 0$

$\tan x = 0$ $\quad \cos x = \frac{\sqrt{3}}{2}$

$x_1 = 0$ $\quad x_2 = \frac{\pi}{6}$ $\quad x_3 = -\frac{\pi}{6}$

⑧ π , $\pi + 6n\pi$, $2\pi + 6n\pi$, $n \in \mathbb{Z}$

⑨ $0.26 + \frac{2n\pi}{3}$

$0.79 + \frac{2n\pi}{3}$, $n \in \mathbb{Z}$

⑩

$\frac{7\pi}{6} + 2n\pi$, $\frac{11\pi}{6} + 2n\pi$, $n \in \mathbb{Z}$

$0.73 + 2n\pi$, $2.41 + 2n\pi$

⑪ $\frac{\pi}{2} + 2n\pi$, $\frac{3\pi}{2} + 2n\pi$, $\frac{\pi}{4} + 2n\pi$, $\frac{5\pi}{4} + 2n\pi$, $n \in \mathbb{Z}$

$\frac{\pi}{2} + n\pi$

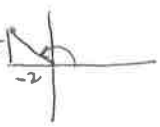
⑫ $\begin{cases} \cos\theta \neq \frac{1}{2} \\ \cos\theta \neq 0 \end{cases}$

⑬ $\frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\sin\theta} + \frac{1}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin\theta} = \frac{1}{\cos^2\theta \sin\theta} = \csc\theta \cdot \sec^2\theta$

⑭ $3\cos(x)$

(15) $\frac{6\sin\theta}{2\sin\theta\cos\theta} = \frac{3}{\cos\theta} = 3\sec\theta$

(17) 

$\sec\theta = -\frac{\sqrt{29}}{2}$

(18) $a = 3$ $\quad y = -3\cos 2\left(x - \frac{\pi}{6}\right) + 2$

$P = \frac{2\pi}{2} = \pi$

$Max = 2+3 = 5$

$Min = 2-3 = -1$

Ch. (7) COMBINATORICS

(1) $30C_5$

(2) $T_4 = {}_6C_3 (3x)^3 (-2)^3$
$= 20 \cdot 27x^3 \cdot (-8)$
$= -4320x^3$

(3) 3360

(4) 181440

(5) 3744

(6) $\frac{10!}{7!} = 720$

(7) $-80x^2y^3$

(8) $n = 7$

(9) $\frac{33!}{5!\,28!}$

(10) $T_6 = {}_{10}C_5 (x)^5 (-y)^5$
$= -252x^5y^5$

(11) a) 4060
b) 24360
c) 1900

(12) 8

(13) 210

(14) 2496

(15) 60

(16) 126

(17) 45

(18) 201058

(19) 1728