

# COMBINATORICS

## SPECS 2001

1. A special combination lock that has 60 numbers on the dial works by turning it first to the right, then to the left, and then to the right, with 3 different selected numbers needed to open the lock. The selection of these 3 numbers is an example of
  - A. a permutation.
  - B. a combination.
  - C. both a combination and a permutation.
  - D. neither a combination nor a permutation.
  
2. Katie wants to colour a rainbow. She knows the seven colours that make up the rainbow, but can't remember the correct order. How many different ways could the colours be arranged assuming each colour is used only once?
  - A. 28
  - B. 128
  - C. 720
  - D. 5 040
  
3. Linda and Sam play a tennis match. The first person to win 2 games wins the match. In how many different ways can a winner be determined?
  - A. 3
  - B. 5
  - C. 6
  - D. 8
  
4. How many 6-digit numbers greater than 800 000 can be made from the digits 1, 1, 5, 5, 5, 8 ?
  - A. 10
  - B. 60
  - C. 64
  - D. 120

5. In how many ways can four colas, three iced teas, and three orange juices be distributed among ten graduates if each graduate is to receive one beverage?

- A. 36
- B. 4 200
- C. 604 800
- D. 3 628 800

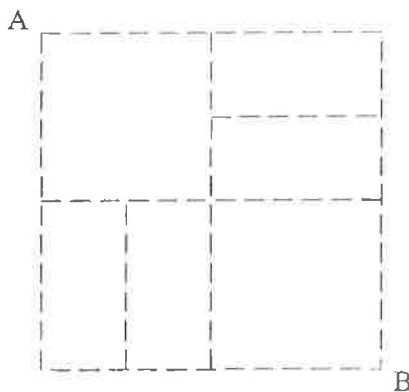
6. Solve for  $n$ :  ${}_nP_2 = 42$

- A. 2
- B. 6
- C. 7
- D. 42

7. Which term in the expansion of  $\left(\frac{1}{2x^2} - x^3\right)^{10}$  is a constant?

- A. 4<sup>th</sup>
- B. 5<sup>th</sup>
- C. 6<sup>th</sup>
- D. 11<sup>th</sup>

~~X~~ Moving only to the right or down, how many different routes exist to get from point A to point B?



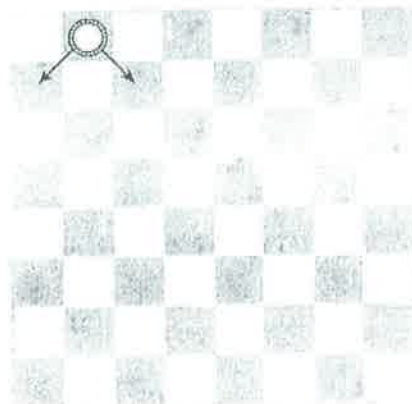
- A. 5
- B. 6
- C. 7
- D. 8

9. What is the 10<sup>th</sup> term of  $\left(2x - \frac{1}{y}\right)^{10}$  ?

10. Numbers are formed on a calculator using seven lines which are either lit or not lit. The diagram below shows the number 8 formed using all 7 lines lit. How many different symbols can be created by lighting one or more of these 7 lines? (Count all the symbols, not just the ones that represent numbers.)



11. A checkerboard is an  $8 \times 8$  game board, as shown below. Game pieces can travel only diagonally on the dark squares, one diagonal square at a time, and only in a downward direction. If a checker is placed as shown, how many possible paths are there for the checker to reach the opposite side of the game board?



### SAMPLE 2001

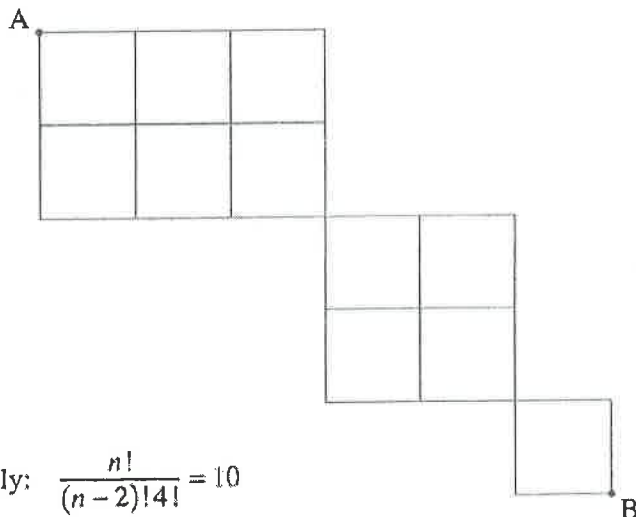
12. How many different committees of 2 people can be selected from 5 people?

- A.  $\frac{5!}{2!}$   
 B.  $\frac{5!}{3!}$   
 C.  $\frac{5!}{2!3!}$   
 D.  $5!$

13. Determine the 5<sup>th</sup> term in the expansion of  $(x - \frac{1}{2}y)^7$ .

- A.  $\frac{35}{8}x^4y^3$   
 B.  $\frac{35}{16}x^3y^4$   
 C.  $-\frac{35}{8}x^4y^3$   
 D.  $-\frac{35}{16}x^3y^4$

14. Moving only to the right or down, how many different paths exist to get from point A to point B?



- A. 22  
 B. 60  
 C. 120  
 D. 144

15. Solve algebraically:  $\frac{n!}{(n-2)!4!} = 10$

## JAN 2002

16. When you play lotto 5-30, you must choose 5 different integers from 1 to 30. How many combinations are possible?

A.  $\frac{30!}{5!25!}$

B.  $\frac{30!}{25!}$

C.  $25!$

D.  $\frac{30!}{5!}$

17. Determine the 4<sup>th</sup> term of  $(x - 2)^6$ .

A.  $120x^2$

B.  $240x^2$

C.  $-160x^3$

D.  $-320x^3$

18. Determine the number of different arrangements of all the letters in APPLEPIE.

A. 3 360

B. 6 720

C. 40 312

D. 40 320

19. Assume a car license plate consists of 7 characters. The first 3 characters can be any of the letters from A to F, but no letter can be repeated. The next 3 characters can be any of the digits from 1 to 9, but no digit can be repeated. The last character can be any of the letters X, Y or Z. An example of this format is: BFA648Y. How many license plates are possible?

A. 5 040

B. 181 440

C. 472 392

D. 4 084 080

20. Suppose you play a game of cards in which only three cards are dealt from a standard 52-card deck. How many ways are there to obtain one pair? (2 cards of the same rank and 1 card of a different rank.)

An example of a hand that contains one pair is 2 jacks and 1 five.



A. 1 014

B. 1 872

C. 3 744

D. 3 900

## APR 2002

21. Determine the 4<sup>th</sup> term in the expansion of  $(x - 2y)^5$ .

A.  $-80x^2y^3$

B.  $-40x^3y^2$

C.  $40x^3y^2$

D.  $80x^2y^3$

22. Solve algebraically:  $\frac{(n-1)!}{(n-3)!} = 30$

## JUN 2002

23. Express  ${}_{33}C_5$  using factorial notation.

A.  $\frac{33!}{5!}$

B.  $\frac{33!}{28!}$

C.  $\frac{33!}{5!28!}$

D.  $28!$

24. Determine the 3<sup>rd</sup> term in the expansion of  $(x - y)^{10}$ .

A.  $-45x^8y^2$

B.  $-120x^7y^3$

C.  $45x^8y^2$

D.  $120x^7y^3$

25. A class has 30 students.

a) How many ways can a committee of 3 people be selected from the class?

b) How many ways can an executive committee consisting of 3 people (president, vice-president, secretary) be selected from the class?

c) If there are 10 boys and 20 girls in the class, how many ways can a committee of 3 people be selected from the class if the committee must contain 1 boy and 2 girls?

## AUG 2002

26. How many different pasta meals can be made from 4 choices of pasta and 2 choices of sauces, if only one pasta and one sauce is selected for each meal?

A. 4

B. 6

C. 8

D. 16

27. A man has 7 different pets and wishes to photograph them 3 at a time arranged in a line. How many different arrangements are possible?

A. 21

B. 35

C. 210

D. 840

28. Determine the 3<sup>rd</sup> term of  $(2x + y)^6$ .

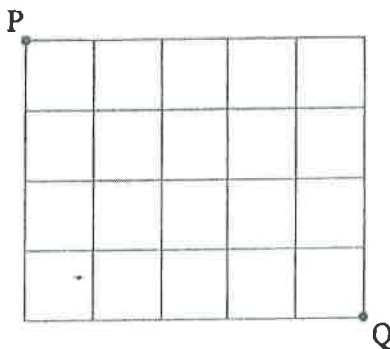
A.  $15x^4y^2$

B.  $240x^4y^2$

C.  $120x^3y^3$

D.  $160x^3y^3$

29. Moving only to the right or down, how many different paths exist to get from point P to point Q?



- A. 120
- B. 126
- C. 180
- D. 480

30. Which expression is equivalent to  ${}_n C_2$  ?

- A.  $n^2 - 2n$
- B.  $n^2 - n$
- C.  $\frac{1}{2}(n^2 - 2n)$
- D.  $\frac{1}{2}(n^2 - n)$

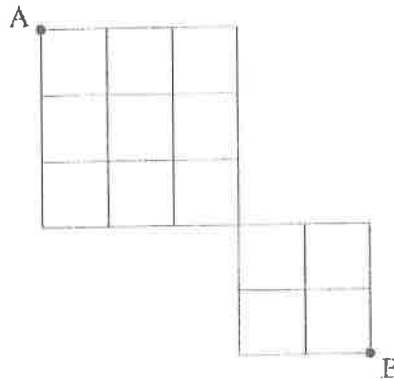
31. Suppose you play a game of cards in which only four cards are dealt from a standard deck of 52 cards. How many ways are there to obtain three of a kind? (3 cards of the same rank and 1 card of a different rank, for example 3 tens and 1 queen.)

- A. 1 872
- B. 2 496
- C. 2 548
- D. 2 704

JAN 2003

32. Moving only to the right or down, how many different paths are there from A to B?

- A. 26
- B. 52
- C. 120
- D. 252



33. Simplify:  $\frac{n(n+1)!}{(n-1)!}$

- A.  $2n!$
- B.  $n!(n^2 + n)$
- C.  $2n$
- D.  $n^3 + n^2$

34. In the expansion of  $(2a - 3b)^6$ , determine the coefficient of the term containing  $a^4b^2$ .

- A. -4 320
- B. 864
- C. 2 160
- D. 2 880

35. A toy box contains 4 different cars and 6 different trucks.

- a) In how many ways can a collection of 5 toys be chosen if the collection must consist of 2 cars and 3 trucks?
- b) In how many ways can a collection of 5 toys be chosen if the collection must consist of at least 3 cars?



JUN 2003

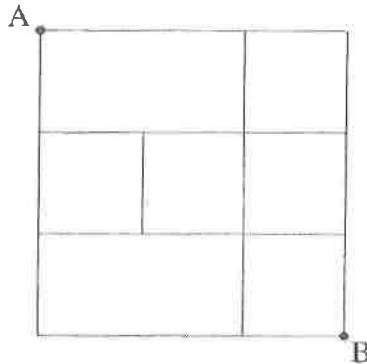
36. How many terms are in the expansion of  $(2x + y)^9$ ?

- A. 8
- B. 9
- C. 10
- D. 11

37. Simplify:  $\frac{(n-2)!}{(n-1)!}$

- A.  $\frac{n-3}{n-1}$
- B.  $n-2$
- C.  $\frac{1}{n-1}$
- D.  $\frac{1}{n(n-1)}$

~~38~~ Moving only to the right or down, how many different routes are there from A to B?



- A. 10
- B. 12
- C. 14
- D. 18

39. There are 7 boys and 5 girls in a group of students.

- a) Calculate the number of ways that a committee of 4 students can be chosen from this group if the committee must have exactly 1 boy.
- b) If the committee of 4 students must have a female president, a male vice-president, and 2 other members chosen from the remaining students, how many ways can such a committee be chosen?

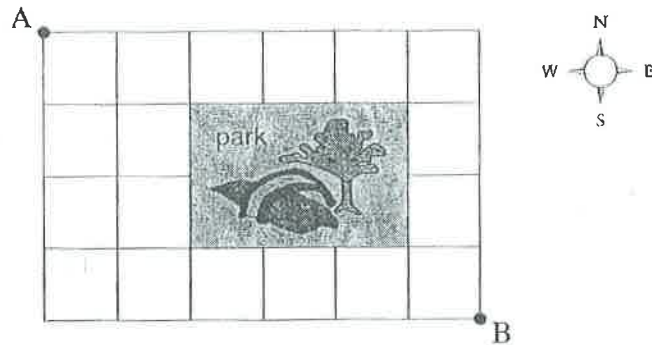
## JAN 2004

40. A couple is planning an evening out. They have a choice of 4 restaurants for dinner, 6 movies following dinner, and 4 coffee establishments for after the movie. How many different ways can they plan the evening if they choose one of each?
- A. 6
  - B. 14
  - C. 48
  - D. 96
41. How many different ways are there to arrange the letters in the word T S A W W A S S E N ?
- A. 25 200
  - B. 151 200
  - C. 302 400
  - D. 3 628 800
42. a) A theatre company of 13 actors consists of 8 men and 5 women. How many different ways are there to choose from the theatre company a group of 7 with exactly 3 men?
- b) A theatre company of 13 actors consists of 8 men and 5 women. How many different ways are there to choose from the theatre company a group of 6 with at least 4 women?

## JUN 2004

43. A student has 7 different textbooks. Which expression gives the number of different ways 4 of these books can be selected and arranged on a shelf?
- A.  $4!$
  - B.  $\frac{7!}{4!}$
  - C.  ${}_7C_4$
  - D.  ${}_7P_4$

44. The diagram below represents a street map. If a person can only travel east or south on the streets, how many different routes are there from A to B?



- A. 60  
B. 68  
C. 80  
D. 200
- The winner of a lottery chooses 4 vehicles from a warehouse that contains 12 different cars, 8 different trucks, and 5 different motorcycles.
45. How many different choices of 4 vehicles are possible?
- A. 480  
B. 570  
C. 12 650  
D. 303 600
46. How many different choices of 4 vehicles are possible if there must be at least one car?
- A. 1 171  
B. 3 432  
C. 9 218  
D. 11 935

## AUG 2005

47. Evaluate:  $\frac{200!}{198!}$
- A. 2
  - B. 200
  - C. 39 800
  - D. infinity
48. A postal code consists of three letters and three digits arranged with a letter first, then a digit, a letter, then a digit, and a letter and a digit. If the first letter must be V, W or X and there are no other restrictions on the other letters or digits, determine how many different postal codes are possible. (An example of a postal code is VON 5Y2.)
- A. 1 259 712
  - B. 1 478 412
  - C. 1 728 000
  - D. 2 028 000
49. Determine the coefficient of the 3<sup>rd</sup> term in the expansion of  $(x + 2y)^7$ .
- A. 21
  - B. 35
  - C. 84
  - D. 140
50. Determine the number of different arrangements of all the letters in the word BALLOON.
- A. 210
  - B. 1260
  - C. 2520
  - D. 5040
51. A class of 34 students consists of 20 girls and 14 boys. How many different committees of 5 girls and 3 boys can be formed from this class?
- A. 2 282 280
  - B. 5 643 456
  - C. 18 643 456
  - D. 40 632 288 320

## AUG 2006

52. How many permutations are there using all of the letters in the word P E P P E R ?

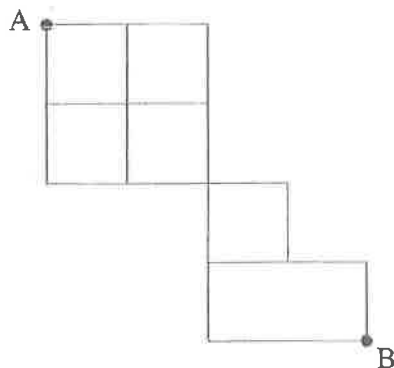
- A. 60
- B. 120
- C. 360
- D. 720

53. In the expansion of  $(x + y)^{10}$ , determine the coefficient of the term containing  $x^8y^2$ .

- A. 9
- B. 10
- C. 36
- D. 45

## SAMPLE 2008

~~54.~~ Determine the number of pathways from point A to point B if only moves to the right and down are permitted.



- A. 18
- B. 19
- C. 23
- D. 47

55. A license plate consists of 3 letters followed by 3 digits. The letters I, O, Q, U, Y and Z are not used. If repetitions of letters and digits are allowed, determine the total number of possible license plates (e.g. ABB603).

- A. 4 924 800
- B. 5 832 000
- C. 8 000 000
- D. 17 576 000

56. Determine the number of different arrangements of all the letters in the word APPLESEED.

- A. 30 240
- B. 60 480
- C. 181 440
- D. 362 880

57. In a standard deck of 52 cards, how many different 5-card hands are there that contain at most 2 face cards?
- A. 652 080
  - B. 844 272
  - C. 1 748 760
  - D. 2 406 768
58. In the expansion of  $(a^2 - b)^4$ , determine the middle term.
- A.  $a^2b^2$
  - B.  $6a^2b^2$
  - C.  $a^4b^2$
  - D.  $6a^4b^2$
59. In how many different ways can a family of 5 people (2 parents and 3 children) sit in a row if a parent must sit on each end of the row?
- A. 6
  - B. 12
  - C. 24
  - D. 120

**JAN 2008**

60. A coach needs to choose an 8-member volleyball team from 10 males and 12 females. If there must be at least 3 of each gender on the team, how many different teams are possible?
- A. 103 950
  - B. 150 480
  - C. 254 430
  - D. 319 770

61. How many even 4-digit whole numbers are there? For example, 1220 is acceptable, but 0678 is not.
- A. 3600  
B. 3645  
C. 4500  
D. 5000
62. An Olympic final race has 7 competitors. In how many possible ways could the gold, silver and bronze medals be awarded?
- A. 21  
B. 35  
C. 210  
D. 5040
63. If  $64a^6$  is the 1st term in the expansion of  $(2a - b)^n$ , determine the coefficient of the 4th term.
- A. -160  
B. -20  
C. 20  
D. 160
64. Each day a student chooses 1 out of 3 beverages in the school cafeteria. Over 10 days he chooses apple juice 3 times, orange juice 3 times and lemonade 4 times. In how many different orders can this occur?
- A. 360  
B. 4 200  
C. 3 628 800  
D. 87 091 200

### ADDITIONAL QUESTIONS

65. Consider the geometric sequence  $1, (a + b), (a + b)^2, \dots$ . Which term of this geometric sequence, when expanded, contains the expression  $35a^4b^3$ ?
- A. 5<sup>th</sup> term  
B. 6<sup>th</sup> term  
C. 7<sup>th</sup> term  
D. 8<sup>th</sup> term
66. Evaluate:  $\sum_{k=1}^4 {}_4C_k$
- A. 1  
B. 4  
C. 15  
D. 16

## 2009 SAMPLE QUESTIONS

67. There are 45 multiple-choice questions on an exam with 4 possible answers for each question. How many different ways are there to complete the test?

- A. 45                      B.  $45 \times 4$                       C.  $45^4$                       D.  $4^{45}$

68. A breakfast special consists of choosing one item from each category in the following menu.

Juice: apple, orange, grapefruit

Toast: white, brown

Eggs: scrambled, fried, poached

Beverage: coffee, tea, milk

How many different breakfast specials are possible?

- A. 11                      B. 48                      C. 54                      D. 96

69. North American area codes are three digit numbers. Before 1995, area codes had the following restrictions: the first digit could not be 0 or 8, the second digit was either 0 or 1, and the third digit was any number from 1 through 9 inclusive. Under these rules, how many different area codes were possible?

- A. 112                      B. 120                      C. 144                      D. 504

~~70.~~ In a particular city, all of the streets run continuously north-south or east-west. The mayor lives 4 blocks east and 5 blocks north of city hall. Determine the number of different routes, 9 blocks in length, that the mayor can take to get to city hall.

- A. 20                      B. 126                      C. 3 024                      D. 15 120

 71. Simplify:  $\frac{6!}{3!2!}$

- A. 1                      B. 20                      C. 60                      D. 120

72. A soccer team played 12 games in a season. They won 6 games, lost 4 games, and tied 2 games. In how many different orders could this have occurred?

- A. 576                      B. 13 860                      C. 9 979 200                      D. 31 933 440



73. Simplify the following expression without using the factorial symbol:  $\frac{(n-2)!(n+1)!}{(n!)^2}$

A.  $\frac{1}{n}$

B.  $\frac{1}{n-1}$

C.  $\frac{n-1}{n(n+1)}$

D.  $\frac{n+1}{n(n-1)}$



74. Solve for  $n$ :  ${}_nC_3 = {}_nP_2$

A. 6

B. 8

C. 1, 8

D. 0, 1, 8

75. Solve:  $\frac{n!}{(n-2)!3!} = 5$

76. There are 2 English books, 3 Chemistry books and 4 Mathematics books to be arranged on a shelf.

a) If all the English books are identical, all the Chemistry books are identical and all the Mathematics books are identical, in how many different ways can they be arranged on the shelf?

b) If the English books, Chemistry books and Mathematics books are all different, in how many different ways can they be arranged on the shelf?

c) If all the English books, Chemistry books and Mathematics books are different, in how many different ways can they be arranged on the shelf if the Chemistry books have to be grouped together?

d) If all the English books, Chemistry books and Mathematics books are different, in how many different ways can they be arranged on the shelf if all the same subject books must be grouped together?

77. Determine the number of different arrangements of all the letters in the word PARALLEL if

a) there are no restrictions.

b) the A's must be together.

c) the first letter must be an A and the last letter must be an A.

d) the first letter must be a vowel.



78. A soccer coach must choose 3 out of 10 players to kick tie-breaking penalty shots. Assuming the coach must designate the order of the 3 players, determine the number of different arrangements she has available.

A.  $\frac{10!}{7!}$

B.  $\frac{10!}{3!}$

C.  $\frac{10!}{3!7!}$

D.  $\frac{10!}{3!3!4!}$

79. A bowl contains an apple, an orange, a plum and a banana. How many different pairs of fruit can be selected from the bowl?

A.  ${}_4P_2$

B.  ${}_2P_4$

C.  ${}_4C_2$

D.  ${}_2C_4$

80. In a standard deck of 52 cards, how many different 4-card hands are there that contain at most one heart?

A. 91 403

B. 118 807

C. 188 474

D. 201 058

81. Assuming that at least one coin is used, how many different sums of money can be made from the following coins: a penny, a nickel, a dime, a quarter and a dollar?

A. 16

B. 31

C. 32

D. 120

82. There are five boys and six girls on a grad committee.

a) In how many ways can a sub-committee of two boys and two girls be selected from the committee?

b) In how many ways can a sub-committee of four people be selected if there must be at least one girl on the sub-committee?

83. How many terms are in the expansion  $\left(2x - \frac{1}{y}\right)^{10}$ ?

A. 9

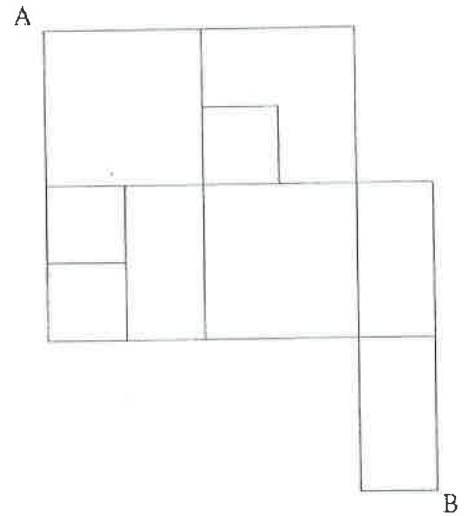
B. 10

C. 11

D. 12

84. Moving only to the right or down, how many different routes exist to get from point A to point B?

- A. 19
- B. 22
- C. 24
- D. 37



85. The 10th term in the expansion of  $\left(x - \frac{1}{2}\right)^n$  is  $-\frac{1001}{256}x^5$ . Determine  $n$ .

- A. 13
- B. 14
- C. 15
- D. 16

86. Determine the 8th term in the expansion of  $(2x - y)^{11}$ .

- A.  $-5280x^4y^7$
- B.  $-2640x^4y^7$
- C.  $1320x^3y^8$
- D.  $990x^3y^8$

## COMBINATORICS

- |    |                                |    |                                       |
|----|--------------------------------|----|---------------------------------------|
| 1  | A                              | 51 | B                                     |
| 2  | D                              | 52 | A                                     |
| 3  | C                              | 53 | D                                     |
| 4  | A                              | 54 | A                                     |
| 5  | B                              | 55 | C                                     |
| 6  | C                              | 56 | A                                     |
| 7  | B                              | 57 | D                                     |
| 8  | D                              | 58 | D                                     |
| 9  | $\frac{-20x}{y^9}$             | 59 | B                                     |
| 10 | 127                            | 60 | C                                     |
| 11 | 69                             | 61 | C                                     |
| 12 | C                              | 62 | C                                     |
| 13 | B                              | 63 | A                                     |
| 14 | C                              | 64 | B                                     |
| 15 | $n = 16$                       | 65 | D                                     |
| 16 | A                              | 66 | C                                     |
| 17 | C                              | 67 | D                                     |
| 18 | A                              | 68 | C                                     |
| 19 | B                              | 69 | C                                     |
| 20 | C                              | 70 | B                                     |
| 21 | A                              | 71 | C                                     |
| 22 | $n = 7$                        | 72 | B                                     |
| 23 | C                              | 73 | D                                     |
| 24 | C                              | 74 | B                                     |
| 25 | a) 4060<br>b) 24360<br>c) 1900 | 75 | $n = 6$                               |
| 26 | C                              | 76 | a) 1260 b) 362880<br>c) 30240 d) 1728 |
| 27 | C                              | 77 | a) 3360 b) 840<br>c) 120 d) 1260      |
| 28 | B                              | 78 | A                                     |
| 29 | B                              | 79 | C                                     |
| 30 | D                              | 80 | D                                     |
| 31 | B                              | 81 | B                                     |
| 32 | C                              | 82 | a) 150 b) 325                         |
| 33 | D                              | 83 | C                                     |
| 34 | C                              | 84 | B                                     |
| 35 | a) 120<br>b) 66                | 85 | B                                     |
| 36 | C                              | 86 | A                                     |
| 37 | C                              |    |                                       |
| 38 | B                              |    |                                       |
| 39 | a) 70<br>b) 1575               |    |                                       |
| 40 | D                              |    |                                       |
| 41 | B                              |    |                                       |
| 42 | a) 280<br>b) 148               |    |                                       |
| 43 | D                              |    |                                       |
| 44 | C                              |    |                                       |
| 45 | C                              |    |                                       |
| 46 | D                              |    |                                       |
| 47 | C                              |    |                                       |
| 48 | D                              |    |                                       |
| 49 | C                              |    |                                       |
| 50 | B                              |    |                                       |