2 - trig ratios in 4 quadrants.docx

Thursday, February 27, 2020 9:36 AM

2 - trig ratios in 4...

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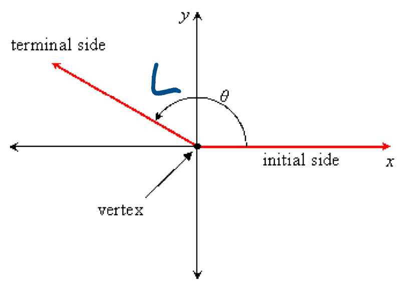
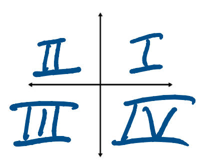
5.2 TRIGONOMETRY Name: ____________ Blk: ________

- **Standard position**:
  - Vertex at the origin of the Cartesian plane
  - Initial arm must coincide with the positive x-axis
  - Positive angles are measured in a counter clockwise direction



- Label the four quadrants of a Cartesian plane:

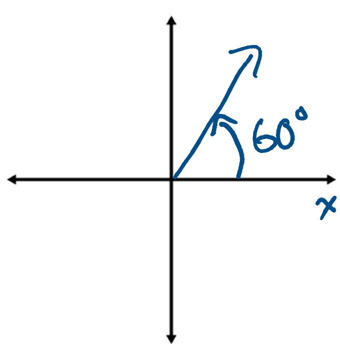


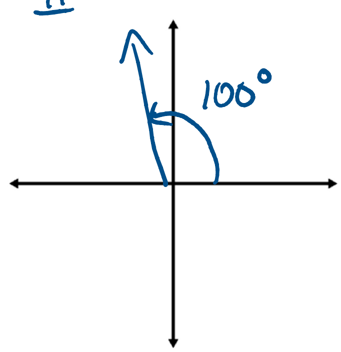
- Try: Draw each angle in standard position and identify the quadrant in which it lies.

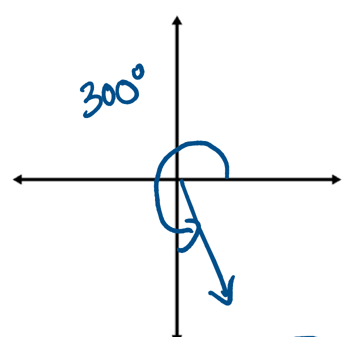
a. 60° I

b. 100° II

c. 300° IV

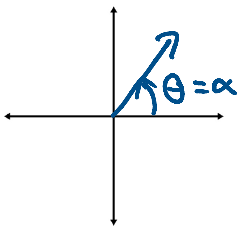
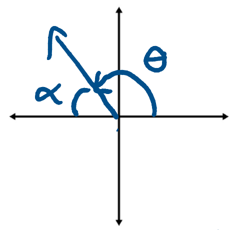
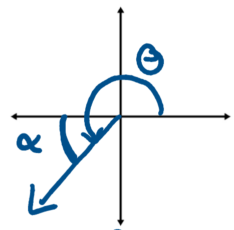
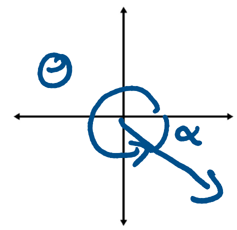




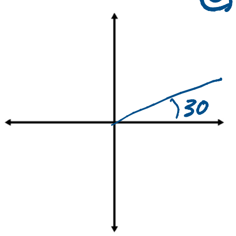
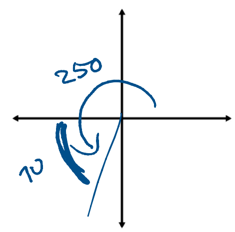
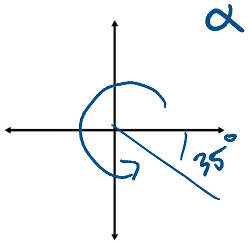
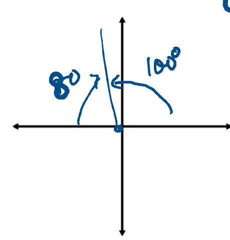


- For each angle in standard, position, there is a corresponding acute angle called the reference angle, which is the acute angle between the terminal arm and the (nearest) x-axis. Thus, any reference angle is between 0° and 90°

α = reference angle

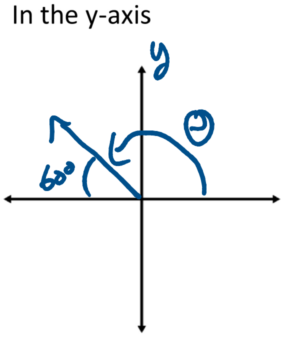
Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
			
$\theta = \alpha$	$\alpha = 180 - \theta$	$\alpha = \theta - 180$	$\alpha = 360 - \theta$

- Try: Draw each angle in standard position, and find the reference angle.

a. 30°  $\theta = \alpha = 30°$	b. 250°  $\alpha = 250 - 180$ $= 70°$
c. 325°  $\alpha = 360 - 325$ $= 35°$	d. 100°  $\alpha = 180 - 100$ $= 80°$

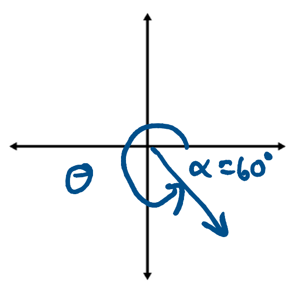
- Try: Determine the angle in standard position when an angle of 60° is reflected

a. In the y-axis



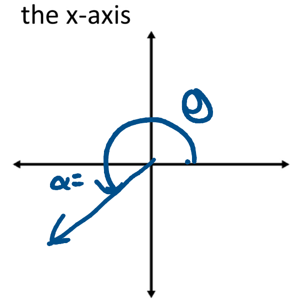
$\theta = 180 - 60$
$= 120°$

b. In the x-axis



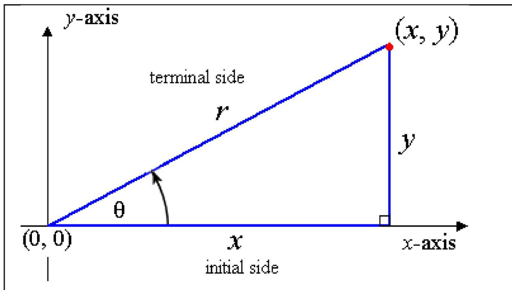
$\theta = 360 - 60$
$= 300°$

c. In the y-axis and then in the x-axis

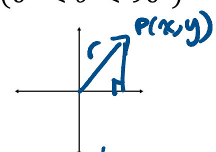
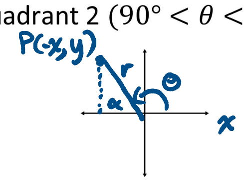
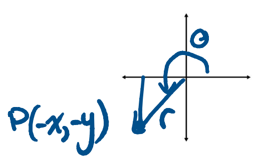
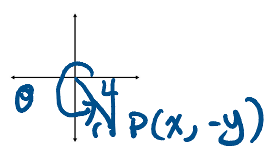


$\theta = 180 + 60$
$= 240°$

- For any angle, θ, the primary trigonometric ratios are:

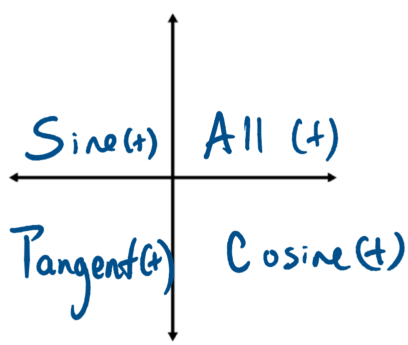
	$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$

- Trigonometry in 4 quadrants:

Quadrant 1 $(0° < \theta < 90°)$  $+ \quad + \quad +$ $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ $\theta = \alpha$	Quadrant 2 $(90° < \theta < 180°)$  $+ \quad - \quad +$ $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{-x}{r}$ $\tan\theta = \frac{y}{-x}$ $\theta = 180 - \alpha$
Quadrant 3 $(180° < \theta < 270°)$  $+$ $\sin\theta = \frac{-y}{r}$ $\cos\theta = \frac{-x}{r}$ $\tan\theta = \frac{-y}{-x} = \boxed{\frac{y}{x}}$ $- \quad -$	Quadrant 4 $(270° < \theta < 360°)$  $\sin\theta = \frac{-y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{-y}{x}$ $- \quad + \quad -$

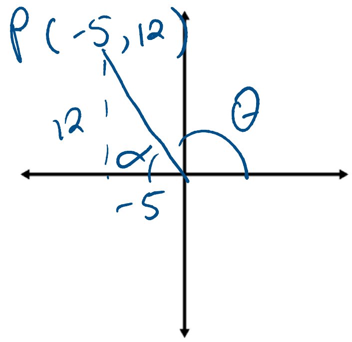
- Here is a way to remember the sign of the trigonometric ratios in each quadrant:

CAST



- Try: The point $(-5,12)$ lies on the terminal arm of an angle, θ, in standard position.

  Determine the exact trigonometric ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$.



$$r = \sqrt{(-5)^2 + 12^2}$$
$$= \sqrt{169}$$
$$= 13$$

$$\sin\theta = \frac{12}{13}$$
$$\cos\theta = \frac{-5}{13}$$
$$\tan\theta = \frac{12}{-5}$$

Determine the measure of θ to the nearest degree.

$$\sin\alpha = \frac{12}{13}$$
$$(\sin^{-1})\sin\alpha = 0.923\ (\sin^{-1})$$
$$\alpha = 67.38^\circ \rightarrow \theta = 180 - \alpha$$
$$= 180 - 67.38$$
$$= 112.62^\circ$$

$$\boxed{\text{Assignment p. 426 \#3-8}}$$

PART II

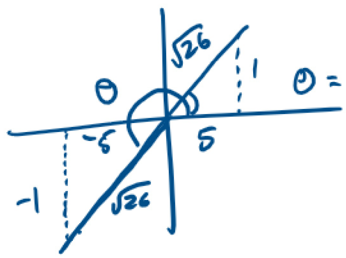
Warm Up:

- Try: Suppose tanθ is an angle in standard position, and $\boxed{\tan\theta = \frac{1}{5}}$. Determine the values of sinθ and cosθ.

$\tan\theta = \frac{y}{x}$

① Find $r = \sqrt{1^2+5^2}$

$r = \sqrt{26}$



Quadrant I

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{26}}$

$\cos\theta = \frac{x}{r} = \frac{5}{\sqrt{26}}$

Quadrant III

$\sin\theta = \frac{-1}{\sqrt{26}}$

$\cos\theta = -\frac{5}{\sqrt{26}}$

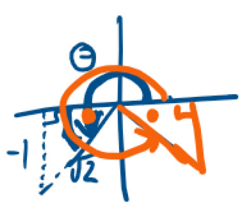
> Solving For Angles:
>
> a. Use the sign ($+$ or $-$) to determine which quadrant the solution(s) is/are in
> b. Solve for the reference angle
> c. Draw a diagram and use the reference angle to find the angle in standard position

> **Your calculator will always give you the angle closest to 0°**

- Try: Determine the standard angle if

where is sine negative? III, IV

a. $\sin\theta = -\frac{1}{\sqrt{2}}, 0° \leq \theta \leq 360°$



Reference angle

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= 45°$

$\theta_{III} = 180 + 45$
$\boxed{= 225°}$

$\theta_{IV} = 360 - 45$
$\boxed{= 315°}$

b. $\cos\theta = 0.5\ \boxed{0° \leq \theta \leq 180°}$

cosine is positive in quadrant I

Reference angle = angle in standard position!

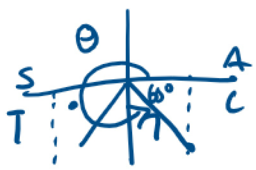
$\cos^{-1}(0.5)$

$= 60°$

$\boxed{\theta = 60°}$

sin is negative in Quadrant ... III, IV

c. $\sin\theta = -\frac{\sqrt{3}}{2}, 0° \le \theta \le 360°$

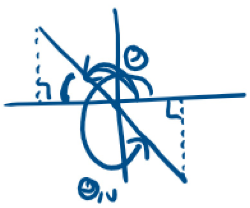


Ref. angle
$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
$= 60°$

$\theta_{III} = 180 + 60$
$= 240°$

$\theta_{IV} = 360 - 60$
$= 300°$

d. $\tan\theta = -0.7565, 0° \le \theta \le 360°$

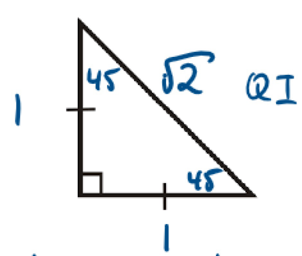
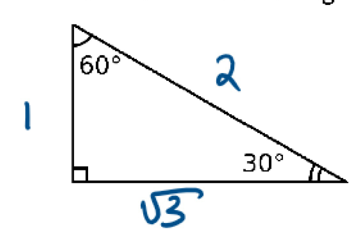


Ref. angle
$\tan^{-1}(0.7565)$
$= 37°$

$\theta_{II} = 180 - 37$
$= 143°$

$\theta_{IV} = 360 - 37$
$= 323°$

- Special Right Triangles:

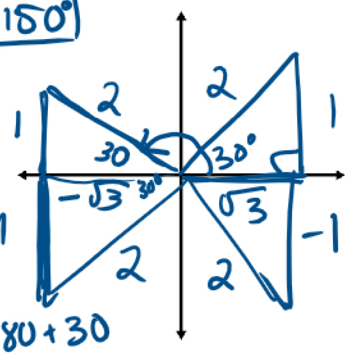
$45° - 45° - 90°$ triangle	$30° - 60° - 90°$ triangle
	
$\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 45° = \frac{1}{1}$ $= 1$	$\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\tan 30° = \frac{1}{\sqrt{3}}$ $\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

- Try: Determine the trigonometric ratios of all the angles that have 30° as a reference angle.

	30°	150°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$

	210°	330°
$\sin\theta$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\cos\theta$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$

$\theta = 180 - 30$
$\boxed{150^\circ}$



$30^\circ = \theta = $ Ref. angle.

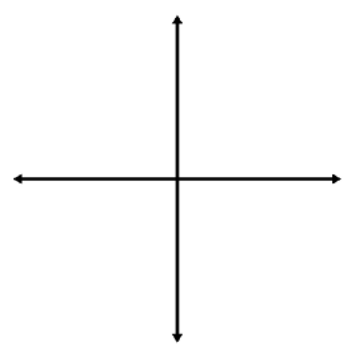
$\theta = 180 + 30$
$= 210$

$\theta = 360 - 30$
$= 330^\circ$

- Fill in the following table with exact values: You need to know these!

	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined.

	300°	315°	330°
$\sin\theta$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$
$\cos\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\tan\theta$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$



Assignment: p.425 – 9, 10, 13–17, 20

Assignment: p.425 ~ 9, 10, 13-17, 20