5-Negative Exponents

Thursday, October 3, 2019 12:41 PM

1.5 POWERS WITH NEGATIVE RATIONAL EXPONENTS Name: _____ Blk: __

Recall: $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$ *and* $x^{-m} = \left(\frac{1}{x}\right)^m$ *or* $\frac{1}{x^m}$ *where* $\frac{1}{x}$ *is the* ***reciprocal*** *of* x

- Two numbers are reciprocals when their **product** is equal to 1
- Example: $4 \cdot \frac{1}{4} = 1$ $\frac{4}{8} \cdot \frac{3}{4} = 1$

Powers with a Negative Integer Exponent and a Rational Base:

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$, where a and b are integers; $a \neq 0, b \neq 0$

Example 1: Evaluate each power

a) 8^{-2} $\left(\frac{1}{8}\right)^2$

$= \frac{1}{8^2}$

$= \boxed{\frac{1}{64}}$

b) $(-2)^{-4}$

$= \frac{1}{(-2)^4}$

$= \boxed{\frac{1}{16}}$

c) $(0.3)^{-3}$

$= \left(\frac{3}{10}\right)^{-3}$

$= \left(\frac{10}{3}\right)^3$

$= \boxed{\frac{1000}{27}}$

d) $\left(\frac{18}{15}\right)^{-2}$ simplify!

$= \left(\frac{15 \div 3}{18 \div 3}\right)^2$

$= \left(\frac{5}{6}\right)^2 = \boxed{\frac{25}{36}}$

Powers with Negative Rational Exponents:

$x^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{x^m}}$ OR $x^{-\frac{m}{n}} = \frac{1}{\left(\sqrt[n]{x}\right)^m}$, where x is a non-zero integer, and m and n are natural numbers; for even values of m and n, $m \neq n$ if $a < 0$.

Example 2: Evaluate each power after writing it as a radical.

a) $(-8)^{-\frac{2}{3}}$

$= \left(\frac{1}{-8}\right)^{2/3}$

$= \frac{1}{\left(\sqrt[3]{(-8)}\right)^2}$

$= \frac{1}{(-2)^2}$

$= \boxed{\frac{1}{4}}$

b) $100^{-2.5}$ $\rightarrow -\frac{5}{2}$

$= 100^{-5/2}$

$= \left(\frac{1}{100}\right)^{5/2}$

$= \frac{1}{\left(\sqrt{100}\right)^5}$

$= \frac{1}{(10)^5}$

$= \boxed{\frac{1}{100000}}$

c) $64^{-\frac{5}{6}}$

$= \left(\frac{1}{64}\right)^{5/6}$

$= \frac{1}{\left(\sqrt[6]{64}\right)^5}$

$= \frac{1}{2^5}$

$= \boxed{\frac{1}{32}}$

d) $-81^{-\frac{3}{4}}$

$= -\left(\frac{1}{81}\right)^{3/4}$

$= -\frac{1}{\left(\sqrt[4]{81}\right)^3}$

$= -\frac{1}{(3)^3}$

$= \boxed{-\frac{1}{27}}$

Powers with Negative Rational Exponents and Rational Bases:

$\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \sqrt[n]{\left(\frac{b}{a}\right)^{m}}$ OR $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\sqrt[n]{\left(\frac{b}{a}\right)}\right)^{m}$, where $\frac{a}{b}$ is a rational number, and $a \neq 0, b \neq 0$, and m and n are natural numbers.

- Basically we are working with all fractions now!

Example 3: Evaluate each power after writing it as a radical

a) $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$

$= \left(\frac{8}{1}\right)^{2/3}$

$= \left(\sqrt[3]{8}\right)^{2}$

$= 2^2$

$= \boxed{4}$

b) $\left(\frac{100}{9}\right)^{-\frac{3}{2}}$

$= \left(\frac{9}{100}\right)^{3/2}$

$= \left(\sqrt{\frac{9}{100}}\right)^{3}$

$= \left(\frac{3}{10}\right)^{3}$

$= \boxed{\frac{27}{1000}}$

c) $\left(\frac{324}{64}\right)^{-\frac{3}{4}}$

$= \left(\frac{64 \div 4}{324 \div 4}\right)^{3/4}$

$= \left(\frac{16}{81}\right)^{3/4}$

$= \left(\sqrt[4]{\frac{16}{81}}\right)^{3}$

$= \left(\frac{2}{3}\right)^{3} = \boxed{\frac{8}{27}}$

d) $\left(-\frac{125}{64}\right)^{-\frac{3}{4}}$

$= \left(-\frac{64}{125}\right)^{3/4}$

$= \left(\sqrt[4]{-\frac{64}{125}}\right)^{3}$ X

= you can't have an even index and a negative number.

Example 4: Application to a word problem

Use the formula $C(t) = A(2)^{-\frac{t}{5}}$ to determine how much caffeine remains in the body after 12 hours when the initial mass of caffeine ingested is 75mg. Give your answer to 1 decimal place.

$C(t) = A(2)^{-t/5}$

$C(12) = 75(2)^{-12/5}$

$= 75\left(\frac{1}{2}\right)^{12/5}$

$= \boxed{14.2\text{mg}}$

$t = 12$ hours $A = 75$mg

$\frac{1}{2}^{\wedge}\left(\frac{12}{5}\right)$ $\frac{1}{2}^{\boxed{y^x}}\left(\frac{12}{5}\right)$

Assignment: In a group answer (WB)

Q # 6a, 7f, 8h, 11e, 12b, 13a, 20 (starts on page 56)

- on a separate paper
- show your steps/work
- Hand in
- you can do a rough copy on whiteboards

HW: Do the rest of WB questions.