Unit 7 Solutions

7.1 Equivalent Rational Expressions

4. Which expressions are rational expressions? Justify your choices.

a) $\frac{x + 3}{5}$ b) $\frac{\sqrt{x} + 2}{4x}$ c) $\frac{a}{7}$

d) $\frac{3^x + 9}{x^2 - 2}$ e) $\frac{3\sqrt[3]{b} + 2b}{16 - b^2}$ f) $\frac{x^2 + 2x - 7}{x + 3}$

Parts a, c, and f are rational expressions because each expression is the quotient of two polynomials. Parts b and e are not rational expressions because they each contain the root of a variable; part d is not a rational expression because it has a variable as an exponent.

5. Identify the non-permissible values of the variable for each rational expression.

a) $\frac{2 - x^2}{x + 5}$

$x + 5 = 0$
$x = -5$
So, $x = -5$ is the non-permissible value.

b) $\frac{x + 1}{(x - 2)(x + 8)}$

$(x - 2)(x + 8) = 0$
$x - 2 = 0$ or $x + 8 = 0$
$x = 2$ or $x = -8$
So, $x = 2$ and $x = -8$ are the non-permissible values.

7. Simplify each rational expression. State the non-permissible values of the variables.

a) $\frac{25mn}{5m}$

The non-permissible value is: $m = 0$

$$\frac{25mn}{5m} = \frac{{}^{5}\cancel{25}\,\cancel{m}\,n}{\cancel{5}\,\cancel{m}}$$
$$= 5n,\ m \neq 0$$

b) $\frac{2x(x-3)}{x-3}$

The non-permissible value is: $x = 3$

$$\frac{2x(x-3)}{x-3} = \frac{2x\,\cancel{(x-3)}}{\cancel{x-3}}$$
$$= 2x,\ x \neq 3$$

c) $\frac{(x-3)(x+4)}{(x+4)(x+6)}$

The non-permissible values are: $x = -4$ and $x = -6$

$$\frac{(x-3)(x+4)}{(x+4)(x+6)}$$
$$= \frac{(x-3)\,\cancel{(x+4)}}{\cancel{(x+4)}\,(x+6)}$$
$$= \frac{x-3}{x+6},\ x \neq -6, -4$$

d) $\frac{3x}{12x(x+5)}$

The non-permissible values are: $x = 0$ and $x = -5$

$$\frac{3x}{12x(x+5)}$$
$$= \frac{\cancel{3x}}{{}_{4}\cancel{12x}(x+5)}$$
$$= \frac{1}{4(x+5)},\ x \neq -5, 0$$

8. Determine the non-permissible values for each rational expression.

a) $\frac{x^2 + 3}{x^2 - x - 20}$

$x^2 - x - 20 = 0$
$(x - 5)(x + 4) = 0$
So, $x = 5$ and $x = -4$ are the non-permissible values.

b) $\frac{3x}{x^2 - 36}$

$x^2 - 36 = 0$
$(x - 6)(x + 6) = 0$
So, $x = -6$ and $x = 6$ are the non-permissible values.

c) $\frac{x(2x - 3)}{4x^2 + 17x - 15}$

$4x^2 + 17x - 15 = 0$
$4x^2 + 20x - 3x - 15 = 0$
$4x(x + 5) - 3(x + 5) = 0$
$(4x - 3)(x + 5) = 0$
So, $x = -5$ and $x = \frac{3}{4}$ are the non-permissible values.

d) $\frac{2x}{12x^2 + 2x}$

$12x^2 + 2x = 0$
$2x(6x + 1) = 0$
So, $x = -\frac{1}{6}$ and $x = 0$ are the non-permissible values.

9. Which of these rational expressions are defined for all real values of x? Explain how you know.

a) $\frac{2x^3 + 3}{6x}$

$x = 0$ is a non-permissible value, so the expression is not defined for all values of x.

b) $\frac{3x + 7}{x^2 - 9}$

$x = 3$ and $x = -3$ are non-permissible values, so the expression is not defined for all values of x.

c) $\frac{3x^2 + 2x - 1}{x^2 + 49}$

Since $x^2 \geq 0$, $x^2 + 49 > 0$
Since the denominator cannot equal 0, the expression is defined for all values of x.

d) $\frac{x^2 + 4}{x^3 + 1}$

$x = -1$ is a non-permissible value, so the expression is not defined for all values of x.

11. **a)** Write each rational expression in simplest form.

i) $\frac{-p^3q^2}{5p^2q^2}$

The non-permissible values are: $p = 0$ and $q = 0$

$$\frac{-p^3q^2}{5p^2q^2} = \frac{-p^{\cancel{3}\,1}\cancel{q^2}}{5\cancel{p^2}\cancel{q^2}}$$

$$= \frac{-p}{5},\ p \neq 0,\ q \neq 0$$

ii) $\frac{4x - 9}{x^2 - 9}$

The non-permissible values are: $x = -3$ and $x = 3$

The numerator and denominator have no common factors, so the expression is in simplest form:

$$\frac{4x - 9}{x^2 - 9},\ x \neq -3, 3$$

iii) $\frac{2x^3 + 4x^2}{6x^2 - 24}$

$$= \frac{2x^2(x + 2)}{6(x^2 - 4)}$$

$$= \frac{2x^2(x + 2)}{6(x - 2)(x + 2)}$$

The non-permissible values are:

$x = -2$ and $x = 2$

$$= \frac{\cancel{2}x^2\cancel{(x + 2)}}{{}_3\cancel{6}(x - 2)\cancel{(x + 2)}}$$

$$= \frac{x^2}{3(x - 2)},\ x \neq -2, 2$$

iv) $\frac{36 - 9x^2}{x^2 - 5x + 6}$

$$= \frac{9(4 - x^2)}{(x - 2)(x - 3)}$$

$$= \frac{9(2 - x)(2 + x)}{(x - 2)(x - 3)}$$

The non-permissible values are:

$x = 2$ and $x = 3$

$$= \frac{-9\cancel{(x - 2)}(2 + x)}{\cancel{(x - 2)}(x - 3)}$$

$$= \frac{-9(2 + x)}{x - 3},\ x \neq 2, 3$$

b) Choose one expression from part a. Explain why the non-permissible values of the given expression and its simplest form are the same.

In part iii, the numerator and denominator of the expression were divided by the common factor $x + 2$. This division is not possible when $x = -2$. So, $x = -2$ must be included in the non-permissible values of the simplified expression.

12. Here is a student's solution for simplifying a rational expression. Identify the error in the solution. Write a correct solution.

$$\frac{3x - 12}{x^2 + x - 20} = \frac{3\cancel{(x - 4)}}{(x + 5)\cancel{(x - 4)}}$$
$$= \frac{3}{x + 5}, x \neq -5$$

The error in the solution is that $x = 4$ should be included as a non-permissible value. Division by $x - 4$ is not possible when $x = 4$.
Correct solution:

$$\frac{3x - 12}{x^2 + x - 20} = \frac{3\cancel{(x - 4)}}{(x + 5)\cancel{(x - 4)}}$$
$$= \frac{3}{(x + 5)}, x \neq -5, 4$$

15. Write each rational expression in simplest form. State the non-permissible values of the variables.

a) $\dfrac{2x^2 - 7xy + 6y^2}{x^4 - 16y^4}$

$$= \frac{2x^2 - 4xy - 3xy + 6y^2}{(x^2 - 4y^2)(x^2 + 4y^2)}$$
$$= \frac{2x(x - 2y) - 3y(x - 2y)}{(x - 2y)(x + 2y)(x^2 + 4y^2)}$$
$$= \frac{(2x - 3y)(x - 2y)}{(x - 2y)(x + 2y)(x^2 + 4y^2)}$$

The non-permissible values are:
$x = 2y$ and $x = -2y$

$$= \frac{(2x - 3y)\cancel{(x - 2y)}}{\cancel{(x - 2y)}(x + 2y)(x^2 + 4y^2)}$$
$$= \frac{2x - 3y}{(x + 2y)(x^2 + 4y^2)}, x \neq 2y, -2y$$

b) $\dfrac{x^4 - 5x^2 + 4}{x^3 + 3x^2 + 2x}$

$$= \frac{(x^2 - 4)(x^2 - 1)}{x(x^2 + 3x + 2)}$$
$$= \frac{(x - 2)(x + 2)(x - 1)(x + 1)}{x(x + 1)(x + 2)}$$

The non-permissible values are:
$x = 0$, $x = -2$, and $x = -1$

$$= \frac{(x - 2)\cancel{(x + 2)}(x - 1)\cancel{(x + 1)}}{x\cancel{(x + 1)}\cancel{(x + 2)}}$$
$$= \frac{(x - 2)(x - 1)}{x}, x \neq -2, -1, 0$$

7.2 Multiplying & Dividing Rational Expressions

4. Simplify each expression.

a) $\frac{2}{n} \div \frac{4}{n}$

Non-permissible value: $n = 0$

$\frac{2}{n} \div \frac{4}{n} = \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}}{\cancel{4}_{2}}$

$= \frac{1}{2}, n \neq 0$

b) $\frac{2b}{5} \div \frac{4}{10b}$

Non-permissible value: $b = 0$

$\frac{2b}{5} \div \frac{4}{10b} = \frac{\cancel{2}b}{\cancel{5}} \cdot \frac{\cancel{10}^{\cancel{2}}b}{\cancel{4}_{\cancel{2}}}$

$= b^2, b \neq 0$

c) $\frac{6}{x} \div 3$

Non-permissible value: $x = 0$

$\frac{6}{x} \div 3 = \frac{\cancel{6}^{2}}{x} \cdot \frac{1}{\cancel{3}}$

$= \frac{2}{x}, x \neq 0$

d) $\frac{d^3}{4} \div \frac{d}{12}$

Non-permissible value: $d = 0$

$\frac{d^3}{4} \div \frac{d}{12} = \frac{d^{\cancel{3}2}}{\cancel{4}} \cdot \frac{\cancel{12}^{3}}{\cancel{d}}$

$= 3d^2, d \neq 0$

5. Simplify each expression.

a) $\frac{m}{5p} \cdot \frac{10p^2}{3m}$

Non-permissible values:
$p = 0$ and $m = 0$

$\frac{m}{5p} \cdot \frac{10p^2}{3m} = \frac{\cancel{m}}{\cancel{5}\cancel{p}} \cdot \frac{\cancel{10}^{2}p^{\cancel{2}}}{3\cancel{m}}$

$= \frac{2p}{3}, p \neq 0, m \neq 0$

b) $\frac{-2ab^2}{5a^3b} \div \frac{-4a}{15ab}$

Non-permissible values:
$a = 0$ and $b = 0$

$\frac{-2ab^2}{5a^3b} \div \frac{-4a}{15ab} = \frac{\cancel{-2}\cancel{a}b^2}{\cancel{5}a^{\cancel{3}2}\cancel{b}} \cdot \frac{\cancel{15}^{3}\cancel{a}\cancel{b}}{_{2}\cancel{-4}\cancel{a}}$

$= \frac{3b^2}{2a^2}, a \neq 0, b \neq 0$

6. Simplify each expression.

a) $\frac{3x}{2(x-3)} \cdot \frac{8(x-3)}{9x^2}$

Non-permissible values:
$x = 0$ and $x = 3$

$= \frac{\cancel{3}\cancel{x}}{\cancel{2}\cancel{(x-3)}} \cdot \frac{\cancel{8}^{4}\cancel{(x-3)}}{_{3}\cancel{9}x^{\cancel{2}}}$

$= \frac{4}{3x}, x \neq 0, 3$

b) $\frac{15(x+5)}{2x^2} \cdot \frac{8x}{5(x+5)^2}$

Non-permissible values:
$x = 0$ and $x = -5$

$= \frac{\cancel{15}^{3}\cancel{(x+5)}}{\cancel{2}x^{\cancel{2}}} \cdot \frac{\cancel{8}^{4}\cancel{x}}{\cancel{5}(x+5)^{\cancel{2}}}$

$= \frac{12}{x(x+5)}, x \neq -5, 0$

c) $\frac{2(x-4)(x+5)}{3(x+1)} \div \frac{4(x+5)}{(x+1)^2}$

Non-permissible values:

$x = -5$ and $x = -1$

$= \frac{\cancel{2}(x-4)\cancel{(x+5)}}{3\cancel{(x+1)}} \cdot \frac{(x+1)^{\cancel{2}}}{{}_{2}\cancel{4}\cancel{(x+5)}}$

$= \frac{(x-4)(x+1)}{6}, x \neq -5, -1$

d) $\frac{10y}{(y-3)^3} \div \frac{y(y+1)}{(y-3)^2}$

Non-permissible values:

$y = 3$, $y = 0$, and $y = -1$

$= \frac{10\cancel{y}}{(y-3)^{\cancel{3}}} \cdot \frac{\cancel{(y-3)^2}}{\cancel{y}(y+1)}$

$= \frac{10}{(y-3)(y+1)}, y \neq -1, 0, 3$

7. Simplify each expression.

a) $\frac{2a+2b}{3a-6b} \cdot \frac{a-2b}{a+b}$

Non-permissible values:

$a = 2b$ and $a = -b$

$\frac{2a+2b}{3a-6b} \cdot \frac{a-2b}{a+b}$

$= \frac{2\cancel{(a+b)}}{3\cancel{(a-2b)}} \cdot \frac{\cancel{a-2b}}{\cancel{a+b}}$

$= \frac{2}{3}, a \neq 2b, -b$

b) $\frac{n^2-4}{2n+1} \cdot \frac{6n+3}{n-2}$

Non-permissible values:

$n = -\frac{1}{2}$ and $n = 2$

$\frac{n^2-4}{2n+1} \cdot \frac{6n+3}{n-2}$

$= \frac{\cancel{(n-2)}(n+2)}{\cancel{2n+1}} \cdot \frac{3\cancel{(2n+1)}}{\cancel{n-2}}$

$= 3(n+2), n \neq -\frac{1}{2}, 2$

c) $\frac{x+5}{x-4} \div \frac{x^2-25}{3x-12}$

$= \frac{x+5}{x-4} \div \frac{(x-5)(x+5)}{3(x-4)}$

Non-permissible values:

$x = -5$, $x = 4$, and $x = 5$

$= \frac{\cancel{x+5}}{\cancel{x-4}} \cdot \frac{3\cancel{(x-4)}}{(x-5)\cancel{(x+5)}}$

$= \frac{3}{x-5}, x \neq -5, 4, 5$

d) $\frac{(b+2)^2}{3b^2-3} \div \frac{2b+4}{1-b}$

$= \frac{(b+2)^2}{3(b^2-1)} \div \frac{2(b+2)}{1-b}$

$= \frac{(b+2)^2}{3(b-1)(b+1)} \div \frac{2(b+2)}{-(b-1)}$

Non-permissible values:

$b = -2$, $b = -1$, and $b = 1$

$= \frac{(b+2)^{\cancel{2}}}{3\cancel{(b-1)}(b+1)} \cdot \frac{-\cancel{(b-1)}}{2\cancel{(b+2)}}$

$= -\frac{b+2}{6(b+1)}, b \neq -2, -1, 1$

8. Simplify each expression.

a) $\frac{x^2 - x - 6}{x + 4} \cdot \frac{x^2 - 16}{x^2 + 2x}$

$= \frac{(x - 3)(x + 2)}{x + 4} \cdot \frac{(x - 4)(x + 4)}{x(x + 2)}$

Non-permissible values:

$x = -4, -2, 0$

$= \frac{(x - 3)\cancel{(x + 2)}}{\cancel{x + 4}} \cdot \frac{(x - 4)\cancel{(x + 4)}}{x\cancel{(x + 2)}}$

$= \frac{(x - 3)(x - 4)}{x}, x \neq -4, -2, 0$

b) $\frac{2x^2 - x - 1}{x^2 + 2x - 3} \cdot \frac{4x^2 + 28x + 48}{2x^2 - 13x - 7}$

$= \frac{(2x + 1)(x - 1)}{(x + 3)(x - 1)} \cdot \frac{4(x^2 + 7x + 12)}{(2x + 1)(x - 7)}$

$= \frac{(2x + 1)(x - 1)}{(x + 3)(x - 1)} \cdot \frac{4(x + 3)(x + 4)}{(2x + 1)(x - 7)}$

Non-permissible values:

$x = -3, -\frac{1}{2}, 1, 7$

$= \frac{\cancel{(2x + 1)}\cancel{(x - 1)}}{\cancel{(x + 3)}\cancel{(x - 1)}} \cdot \frac{4\cancel{(x + 3)}(x + 4)}{\cancel{(2x + 1)}(x - 7)}$

$= \frac{4(x + 4)}{x - 7}, x \neq -3, -\frac{1}{2}, 1, 7$

c) $\frac{m^2 - m - 12}{m + 3} \div \frac{3m - 12}{m^2 - 9}$

$= \frac{(m - 4)(m + 3)}{m + 3} \div \frac{3(m - 4)}{(m - 3)(m + 3)}$

Non-permissible values:

$m = -3, 3, 4$

$= \frac{\cancel{(m - 4)}\cancel{(m + 3)}}{\cancel{m + 3}} \cdot \frac{(m - 3)(m + 3)}{3\cancel{(m - 4)}}$

$= \frac{(m - 3)(m + 3)}{3}, m \neq -3, 3, 4$

d) $\frac{2s^2 - 7s - 4}{6s^2 - 5s - 6} \div \frac{4s^2 + 4s + 1}{12s + 8}$

$= \frac{(2s + 1)(s - 4)}{(3s + 2)(2s - 3)} \div \frac{(2s + 1)(2s + 1)}{4(3s + 2)}$

Non-permissible values:

$s = -\frac{2}{3}, -\frac{1}{2}, \frac{3}{2}$

$= \frac{\cancel{(2s + 1)}(s - 4)}{\cancel{(3s + 2)}(2s - 3)} \cdot \frac{4\cancel{(3s + 2)}}{(2s + 1)\cancel{(2s + 1)}}$

$= \frac{4(s - 4)}{(2s - 3)(2s + 1)}, s \neq -\frac{2}{3}, -\frac{1}{2}, \frac{3}{2}$

11. Simplify each expression.

a) $\frac{3x^3y}{18x^3} \cdot \frac{2y^2}{4xy^5} \cdot \frac{8x^3y^2}{12xy}$

Non-permissible values:

$x = 0, y = 0$

$= \frac{\cancel{3}\cancel{x^3}\cancel{y}}{{}_{3}\cancel{18}\cancel{x^3}} \cdot \frac{\cancel{2}\cancel{y^2}}{\cancel{4}\cancel{x}y^{\cancel{5}}} \cdot \frac{{}^{\cancel{4}}\cancel{8}x^{\cancel{3}}\cancel{y^2}}{{}_{6}\cancel{12}\cancel{x}\cancel{y}}$

$= \frac{x}{18y}, x \neq 0, y \neq 0$

b) $\frac{3ab}{4ab^3} \cdot \frac{8b^2}{27a^5b^3} \div \frac{2a^2b^2}{6ab^3}$

Non-permissible values:

$a = 0, b = 0$

$= \frac{\cancel{3}\cancel{a}\cancel{b}}{\cancel{4}\cancel{a}\cancel{b^3}} \cdot \frac{\cancel{8}\cancel{b^2}}{{}_{3}\cancel{27}a^5b^{\cancel{2}}} \cdot \frac{{}^{2}\cancel{6}\cancel{a}\cancel{b^3}}{\cancel{2}a^{\cancel{2}}\cancel{b^2}}$

$= \frac{2}{3a^4b^2}, a \neq 0, b \neq 0$

12. Simplify each expression.

a) $\dfrac{x+3}{x^2-2x-15} \cdot \dfrac{2x-10}{2x^2+15x+7} \div \dfrac{2x-4}{x^2+5x-14}$

$= \dfrac{x+3}{(x-5)(x+3)} \cdot \dfrac{2(x-5)}{(2x+1)(x+7)} \div \dfrac{2(x-2)}{(x+7)(x-2)}$

Non-permissible values: $x = -7, -3, -\frac{1}{2}, 2, 5$

$= \dfrac{\cancel{x+3}}{\cancel{(x-5)}\cancel{(x+3)}} \cdot \dfrac{\cancel{2}\cancel{(x-5)}}{(2x+1)\cancel{(x+7)}} \cdot \dfrac{\cancel{(x+7)}\cancel{(x-2)}}{\cancel{2}\cancel{(x-2)}}$

$= \dfrac{1}{2x+1}, x \neq -7, -3, -\frac{1}{2}, 2, 5$

b) $\dfrac{x^2-x-56}{x+8} \div \dfrac{x^2+14x+49}{x^2-6x-16} \cdot \dfrac{3x+24}{x^2-16x+64}$

$= \dfrac{(x-8)(x+7)}{x+8} \div \dfrac{(x+7)(x+7)}{(x-8)(x+2)} \cdot \dfrac{3(x+8)}{(x-8)(x-8)}$

Non-permissible values: $x = -8, -7, -2, 8$

$= \dfrac{\cancel{(x-8)}\cancel{(x+7)}}{\cancel{x+8}} \cdot \dfrac{\cancel{(x-8)}(x+2)}{\cancel{(x+7)}(x+7)} \cdot \dfrac{3\cancel{(x+8)}}{\cancel{(x-8)}\cancel{(x-8)}}$

$= \dfrac{3(x+2)}{x+7}, x \neq -8, -7, -2, 8$

c) $\dfrac{x+4y}{x-5y} \cdot \dfrac{x^2-25y^2}{x^2-16y^2} \div \dfrac{x+5y}{x-4y}$

$= \dfrac{x+4y}{x-5y} \cdot \dfrac{(x-5y)(x+5y)}{(x-4y)(x+4y)} \div \dfrac{x+5y}{x-4y}$

Non-permissible values: $x = -5y, -4y, 4y, 5y$

$= \dfrac{\cancel{x+4y}}{\cancel{x-5y}} \cdot \dfrac{\cancel{(x-5y)}\cancel{(x+5y)}}{\cancel{(x-4y)}\cancel{(x+4y)}} \cdot \dfrac{\cancel{x-4y}}{\cancel{x+5y}}$

$= 1, x \neq -5y, -4y, 4y, 5y$

d) $\dfrac{(5c+6d)^2}{3c-4d} \cdot \dfrac{9c^2-16d^2}{25c^2-36d^2} \div \dfrac{3c+4d}{5c-6d}$

$= \dfrac{(5c+6d)^2}{3c-4d} \cdot \dfrac{(3c-4d)(3c+4d)}{(5c-6d)(5c+6d)} \div \dfrac{3c+4d}{5c-6d}$

Non-permissible values: $c = -\frac{4d}{3}, -\frac{6d}{5}, \frac{6d}{5}, \frac{4d}{3}$

$= \dfrac{(5c+6d)^{\cancel{2}}}{\cancel{3c-4d}} \cdot \dfrac{\cancel{(3c-4d)}\cancel{(3c+4d)}}{\cancel{(5c-6d)}\cancel{(5c+6d)}} \cdot \dfrac{\cancel{5c-6d}}{\cancel{3c+4d}}$

$= 5c+6d, c \neq -\frac{4d}{3}, -\frac{6d}{5}, \frac{6d}{5}, \frac{4d}{3}$

15. Simplify: $\dfrac{1 - \frac{2}{a}}{1 - \frac{4}{a^2}}$

$$= \frac{\frac{a - 2}{a}}{\frac{a^2 - 4}{a^2}}$$

$$= \frac{a - 2}{a} \cdot \frac{a^2}{a^2 - 4}$$

$$= \frac{\cancel{a - 2}}{\cancel{a}} \cdot \frac{a^{\cancel{2}}}{\cancel{(a - 2)}(a + 2)}$$ Non-permissible values: $a = -2, 0, 2$

$$= \frac{a}{a + 2}, a \neq -2, 0, 2$$

7.3 Adding and Subtracting Rational Expressions with Monomials

3. Simplify.

a) $\frac{5}{a} + \frac{3}{a}$

$= \frac{8}{a}, a \neq 0$

c) $\dfrac{7e}{e + 2} - \dfrac{3e}{e + 2}$

$= \dfrac{4e}{e + 2}, e \neq -2$

5. Simplify.

a) $\frac{4}{a} - \frac{2}{3}$

Non-permissible value: $a = 0$

Common denominator: $3a$

$= \frac{4}{a} \cdot \frac{3}{3} - \frac{2}{3} \cdot \frac{a}{a}$

$= \frac{12}{3a} - \frac{2a}{3a}$

$= \frac{12 - 2a}{3a}, a \neq 0$

b) $\frac{4}{3x} + \frac{1}{5x}$

Non-permissible value: $x = 0$

Common denominator: $15x$

$= \frac{4}{3x} \cdot \frac{5}{5} + \frac{1}{5x} \cdot \frac{3}{3}$

$= \frac{20}{15x} + \frac{3}{15x}$

$= \frac{23}{15x}, x \neq 0$

6. Simplify.

a) $\frac{1}{3a} + \frac{2a}{5a^2}$

Non-permissible value: $a = 0$

Common denominator: $15a^2$

$= \frac{1}{3a} \cdot \frac{5a}{5a} + \frac{2a}{5a^2} \cdot \frac{3}{3}$

$= \frac{5a}{15a^2} + \frac{6a}{15a^2}$

$= \frac{11\cancel{a}}{15a^{\cancel{2}}}$

$= \frac{11}{15a}, a \neq 0$

b) $4 - \frac{1}{x}$

Non-permissible value: $x = 0$

Common denominator: x

$= \frac{4}{1} \cdot \frac{x}{x} - \frac{1}{x}$

$= \frac{4x}{x} - \frac{1}{x}$

$= \frac{4x - 1}{x}, x \neq 0$

7. Simplify.

a) $\frac{2}{xz^2} - \frac{5}{x^2z}$

Non-permissible values:
$x = 0, z = 0$
Common denominator: x^2z^2

$= \frac{2}{xz^2} \cdot \frac{x}{x} - \frac{5}{x^2z} \cdot \frac{z}{z}$

$= \frac{2x}{x^2z^2} - \frac{5z}{x^2z^2}$

$= \frac{2x - 5z}{x^2z^2}, x \neq 0, z \neq 0$

b) $\frac{3}{r^2s^2} + \frac{2}{rs^3}$

Non-permissible values:
$r = 0, s = 0$
Common denominator: r^2s^3

$= \frac{3}{r^2s^2} \cdot \frac{s}{s} + \frac{2}{rs^3} \cdot \frac{r}{r}$

$= \frac{3s}{r^2s^3} + \frac{2r}{r^2s^3}$

$= \frac{3s + 2r}{r^2s^3}, r \neq 0, s \neq 0$

c) $\frac{1}{x} + \frac{7}{6y}$

Non-permissible values:
$x = 0, y = 0$
Common denominator: $6xy$

$= \frac{1}{x} \cdot \frac{6y}{6y} + \frac{7}{6y} \cdot \frac{x}{x}$

$= \frac{6y}{6xy} + \frac{7x}{6xy}$

$= \frac{6y + 7x}{6xy}, x \neq 0, y \neq 0$

d) $\frac{2}{3e^5f^2} + \frac{3}{5ef^3}$

Non-permissible values:
$e = 0, f = 0$
Common denominator: $15e^5f^3$

$= \frac{2}{3e^5f^2} \cdot \frac{5f}{5f} + \frac{3}{5ef^3} \cdot \frac{3e^4}{3e^4}$

$= \frac{10f}{15e^5f^3} + \frac{9e^4}{15e^5f^3}$

$= \frac{10f + 9e^4}{15e^5f^3}, e \neq 0, f \neq 0$

8. Simplify.

a) $\frac{2x - 3}{x} + \frac{x - 1}{3x}$

Non-permissible value: $x = 0$
Common denominator: $3x$

$= \frac{2x - 3}{x} \cdot \frac{3}{3} + \frac{x - 1}{3x}$

$= \frac{3(2x - 3)}{3x} + \frac{x - 1}{3x}$

$= \frac{6x - 9 + x - 1}{3x}$

$= \frac{7x - 10}{3x}, x \neq 0$

b) $\frac{2p + 4}{2p} + \frac{3p + 1}{3p^2}$

Non-permissible value: $p = 0$
Common denominator: $6p^2$

$= \frac{2p + 4}{2p} \cdot \frac{3p}{3p} + \frac{3p + 1}{3p^2} \cdot \frac{2}{2}$

$= \frac{3p(2p + 4)}{6p^2} + \frac{2(3p + 1)}{6p^2}$

$= \frac{6p^2 + 12p + 6p + 2}{6p^2}$

$= \frac{6p^2 + 18p + 2}{6p^2}$

$= \frac{\cancel{2}(3p^2 + 9p + 1)}{{}_3\cancel{6}p^2}$

$= \frac{3p^2 + 9p + 1}{3p^2}, p \neq 0$

c) $\frac{3t - 4}{t} - \frac{2t - 7}{8}$

Non-permissible value: $t = 0$
Common denominator: $8t$

$= \frac{3t - 4}{t} \cdot \frac{8}{8} - \frac{2t - 7}{8} \cdot \frac{t}{t}$

$= \frac{8(3t - 4)}{8t} - \frac{t(2t - 7)}{8t}$

$= \frac{24t - 32 - 2t^2 + 7t}{8t}$

$= \frac{-2t^2 + 31t - 32}{8t}, t \neq 0$

d) $\frac{2q - 3}{5q^2} - \frac{1 - q}{4q^3}$

Non-permissible value: $q = 0$
Common denominator: $20q^3$

$= \frac{2q - 3}{5q^2} \cdot \frac{4q}{4q} - \frac{1 - q}{4q^3} \cdot \frac{5}{5}$

$= \frac{4q(2q - 3)}{20q^3} - \frac{5(1 - q)}{20q^3}$

$= \frac{8q^2 - 12q - 5 + 5q}{20q^3}$

$= \frac{8q^2 - 7q - 5}{20q^3}, q \neq 0$

9. Simplify.

a) $\frac{a + b}{5a} - \frac{b - a}{15b}$

Non-permissible values:
$a = 0, b = 0$
Common denominator: $15ab$

$= \frac{a + b}{5a} \cdot \frac{3b}{3b} - \frac{b - a}{15b} \cdot \frac{a}{a}$

$= \frac{3b(a + b)}{15ab} - \frac{a(b - a)}{15ab}$

$= \frac{3ab + 3b^2 - ab + a^2}{15ab}$

$= \frac{3b^2 + 2ab + a^2}{15ab}, a \neq 0, b \neq 0$

b) $\frac{2x - y}{x^2y} + \frac{x + y}{xy^2}$

Non-permissible values:
$x = 0, y = 0$
Common denominator: x^2y^2

$= \frac{2x - y}{x^2y} \cdot \frac{y}{y} + \frac{x + y}{xy^2} \cdot \frac{x}{x}$

$= \frac{y(2x - y)}{x^2y^2} + \frac{x(x + y)}{x^2y^2}$

$= \frac{2xy - y^2 + x^2 + xy}{x^2y^2}$

$= \frac{x^2 + 3xy - y^2}{x^2y^2}, x \neq 0, y \neq 0$

c) $\frac{11j - 7}{6j^2} - \frac{k + 2}{7k}$

Non-permissible values:
$j = 0, k = 0$
Common denominator: $42j^2k$

$= \frac{11j - 7}{6j^2} \cdot \frac{7k}{7k} - \frac{k + 2}{7k} \cdot \frac{6j^2}{6j^2}$

$= \frac{7k(11j - 7)}{42j^2k} - \frac{6j^2(k + 2)}{42j^2k}$

$= \frac{77jk - 49k - 6j^2k - 12j^2}{42j^2k},$

$j \neq 0, k \neq 0$

d) $\frac{b + 1}{7bc} - \frac{b^2 - 1}{8b^3}$

Non-permissible values:
$b = 0, c = 0$
Common denominator: $56b^3c$

$= \frac{b + 1}{7bc} \cdot \frac{8b^2}{8b^2} - \frac{b^2 - 1}{8b^3} \cdot \frac{7c}{7c}$

$= \frac{8b^2(b + 1)}{56b^3c} - \frac{7c(b^2 - 1)}{56b^3c}$

$= \frac{8b^3 + 8b^2 - 7b^2c + 7c}{56b^3c},$

$b \neq 0, c \neq 0$

12. Simplify.

a) $\frac{x^2 - 3}{2x} + x - 2$

Non-permissible value: $x = 0$

Common denominator: $2x$

$= \frac{x^2 - 3}{2x} + \frac{x}{1} \cdot \frac{2x}{2x} - \frac{2}{1} \cdot \frac{2x}{2x}$

$= \frac{x^2 - 3}{2x} + \frac{2x^2}{2x} - \frac{4x}{2x}$

$= \frac{3x^2 - 4x - 3}{2x}, x \neq 0$

b) $4w + \frac{1}{v} - \frac{2w - 3}{w^2}$

Non-permissible values: $v = 0, w = 0$

Common denominator: vw^2

$= \frac{4w}{1} \cdot \frac{vw^2}{vw^2} + \frac{1}{v} \cdot \frac{w^2}{w^2} - \frac{(2w - 3)}{w^2} \cdot \frac{v}{v}$

$= \frac{4vw^3}{vw^2} + \frac{w^2}{vw^2} - \frac{v(2w - 3)}{vw^2}$

$= \frac{4vw^3 + w^2 - 2vw + 3v}{vw^2}, v \neq 0, w \neq 0$

c) $\frac{x + y}{y} + \frac{x - y}{x} - 4xy$

Non-permissible values:

$x = 0$ and $y = 0$

Common denominator: xy

$= \frac{(x + y)}{y} \cdot \frac{x}{x}$

$+ \frac{(x - y)}{x} \cdot \frac{y}{y} - \frac{4xy}{1} \cdot \frac{xy}{xy}$

$= \frac{x(x + y)}{xy} + \frac{y(x - y)}{xy} - \frac{4x^2y^2}{xy}$

$= \frac{x^2 + xy + xy - y^2 - 4x^2y^2}{xy}$

$= \frac{x^2 + 2xy - y^2 - 4x^2y^2}{xy},$

$x \neq 0, y \neq 0$

d) $\frac{-2 + t}{6t} + 3 - \frac{2t + 3}{5}$

Non-permissible value:

$t = 0$

Common denominator: $30t$

$= \frac{(-2 + t)}{6t} \cdot \frac{5}{5} + \frac{3}{1} \cdot \frac{30t}{30t} - \frac{(2t + 3)}{5} \cdot \frac{6t}{6t}$

$= \frac{5(-2 + t) + 90t - 6t(2t + 3)}{30t}$

$= \frac{-10 + 5t + 90t - 12t^2 - 18t}{30t}$

$= \frac{-12t^2 + 77t - 10}{30t}, t \neq 0$

13. Simplify.

a) $\frac{3}{2a} + \frac{5a^3}{3ab^3} \cdot \frac{12b^2}{10a^2}$

Non-permissible values:

$a = 0, b = 0$

$= \frac{3}{2a} + \frac{\cancel{5}\,\cancel{a^3}}{\cancel{3}\,\cancel{a}\,b^{\cancel{3}}} \cdot \frac{^{2}\cancel{12}\,\cancel{b^2}}{\cancel{10}\,\cancel{a^2}}$

$= \frac{3}{2a} + \frac{2}{b}$

Common denominator: $2ab$

$= \frac{3}{2a} \cdot \frac{b}{b} + \frac{2}{b} \cdot \frac{2a}{2a}$

$= \frac{3b + 4a}{2ab}, a \neq 0, b \neq 0$

b) $\frac{6x}{2(x + 3)} \div \frac{4x^2}{5(x + 3)} - \frac{x}{2}$

Non-permissible values:

$x = -3, x = 0$

$= \frac{^{3}\cancel{6}\,\cancel{x}}{\cancel{2}\,\cancel{(x + 3)}} \cdot \frac{5\,\cancel{(x + 3)}}{4x^{\cancel{2}}} - \frac{x}{2}$

$= \frac{15}{4x} - \frac{x}{2}$

Common denominator: $4x$

$= \frac{15}{4x} - \frac{x}{2} \cdot \frac{2x}{2x}$

$= \frac{15 - 2x^2}{4x}, x \neq -3, 0$

7.4 Adding & Subtracting Rational Expressions with Binomial & Trinomial Denominators

3. Simplify.

a) $\frac{4x}{x+5} + \frac{6x}{x+5}$

$= \frac{10x}{x+5}, x \neq -5$

b) $\frac{n+1}{n-2} + \frac{n-4}{n-2}$

$= \frac{n+1+n-4}{n-2}$

$= \frac{2n-3}{n-2}, n \neq 2$

c) $\frac{2b}{3b^2+1} - \frac{3b}{3b^2+1}$

$= \frac{-b}{3b^2+1}$

d) $\frac{2c-9}{c^2+2c+1} - \frac{4c-9}{c^2+2c+1}$

$= \frac{2c-9-4c+9}{c^2+2c+1}$

$= \frac{-2c}{(c+1)^2}, c \neq -1$

5. Simplify.

a) $\frac{1}{x-2} - \frac{2}{x+2}$

Common denominator:
$(x-2)(x+2)$

$= \frac{1}{(x-2)} \cdot \frac{(x+2)}{(x+2)}$

$- \frac{2}{(x+2)} \cdot \frac{(x-2)}{(x-2)}$

$= \frac{x+2}{(x-2)(x+2)} - \frac{2x-4}{(x-2)(x+2)}$

$= \frac{-x+6}{(x-2)(x+2)}, x \neq -2, 2$

b) $\frac{6}{a-3} + \frac{2}{a+7}$

Common denominator:
$(a-3)(a+7)$

$= \frac{6}{(a-3)} \cdot \frac{(a+7)}{(a+7)}$

$+ \frac{2}{(a+7)} \cdot \frac{(a-3)}{(a-3)}$

$= \frac{6a+42}{(a-3)(a+7)} + \frac{2a-6}{(a-3)(a+7)}$

$= \frac{8a+36}{(a-3)(a+7)}$

$= \frac{4(2a+9)}{(a-3)(a+7)}, a \neq -7, 3$

c) $\frac{7}{b+9} - \frac{4}{b-2}$

Common denominator:
$(b+9)(b-2)$

$$= \frac{7}{(b+9)} \cdot \frac{(b-2)}{(b-2)} - \frac{4}{(b-2)} \cdot \frac{(b+9)}{(b+9)}$$

$$= \frac{7b-14}{(b+9)(b-2)} - \frac{4b+36}{(b+9)(b-2)}$$

$$= \frac{3b-50}{(b+9)(b-2)}, b \neq -9, 2$$

d) $\frac{-5}{w} + \frac{2}{w-4}$

Common denominator:
$w(w-4)$

$$= \frac{-5}{w} \cdot \frac{(w-4)}{(w-4)} + \frac{2}{(w-4)} \cdot \frac{w}{w}$$

$$= \frac{-5w+20}{w(w-4)} + \frac{2w}{w(w-4)}$$

$$= \frac{-3w+20}{w(w-4)}, w \neq 0, 4$$

6. Simplify.

a) $\frac{6}{r-4} + \frac{r+5}{4-r}$

$$= \frac{6}{r-4} + \frac{r+5}{-(r-4)}$$

$$= \frac{6}{r-4} - \frac{r+5}{r-4}$$

$$= \frac{-r+1}{r-4}, r \neq 4$$

b) $\frac{t+3}{t-4} - \frac{t-2}{t-5}$

Common denominator:
$(t-4)(t-5)$

$$= \frac{(t+3)}{(t-4)} \cdot \frac{(t-5)}{(t-5)} - \frac{(t-2)}{(t-5)} \cdot \frac{(t-4)}{(t-4)}$$

$$= \frac{t^2-2t-15}{(t-4)(t-5)} - \frac{t^2-6t+8}{(t-4)(t-5)}$$

$$= \frac{4t-23}{(t-4)(t-5)}, t \neq 4, 5$$

7. Simplify.

a) $\frac{8}{6x + 9} + \frac{3}{4x - 4}$

$= \frac{8}{3(2x + 3)} + \frac{3}{4(x - 1)}$

Common denominator:

$12(2x + 3)(x - 1)$

$= \frac{8}{3(2x + 3)} \cdot \frac{4(x - 1)}{4(x - 1)}$

$+ \frac{3}{4(x - 1)} \cdot \frac{3(2x + 3)}{3(2x + 3)}$

$= \frac{32(x - 1)}{12(2x + 3)(x - 1)}$

$+ \frac{9(2x + 3)}{12(2x + 3)(x - 1)}$

$= \frac{32x - 32 + 18x + 27}{12(2x + 3)(x - 1)}$

$= \frac{50x - 5}{12(2x + 3)(x - 1)}$

$= \frac{5(10x - 1)}{12(2x + 3)(x - 1)}, x \neq -\frac{3}{2}, 1$

b) $\frac{10}{4k^2 - 4} - \frac{2}{5 - 5k}$

$= \frac{10}{4(k^2 - 1)} - \frac{2}{5(1 - k)}$

$= \frac{10}{4(k - 1)(k + 1)} + \frac{2}{5(k - 1)}$

Common denominator:

$20(k - 1)(k + 1)$

$= \frac{10}{4(k - 1)(k + 1)} \cdot \frac{5}{5} + \frac{2}{5(k - 1)} \cdot \frac{4(k + 1)}{4(k + 1)}$

$= \frac{50}{20(k - 1)(k + 1)} + \frac{8k + 8}{20(k - 1)(k + 1)}$

$= \frac{58 + 8k}{20(k - 1)(k + 1)}$

$= \frac{2(29 + 4k)}{20(k - 1)(k + 1)}$

$= \frac{29 + 4k}{10(k - 1)(k + 1)}, k \neq -1, 1$

10. Simplify.

a) $3 + \frac{5x^2 + 8}{x^2 - 2x - 8}$

$= \frac{3}{1} + \frac{5x^2 + 8}{(x - 4)(x + 2)}$

Common denominator:

$(x - 4)(x + 2)$

$= \frac{3}{1} \cdot \frac{(x - 4)(x + 2)}{(x - 4)(x + 2)}$

$+ \frac{5x^2 + 8}{(x - 4)(x + 2)}$

$= \frac{3(x^2 - 2x - 8)}{(x - 4)(x + 2)}$

$+ \frac{5x^2 + 8}{(x - 4)(x + 2)}$

$= \frac{3x^2 - 6x - 24 + 5x^2 + 8}{(x - 4)(x + 2)}$

$= \frac{8x^2 - 6x - 16}{(x - 4)(x + 2)}$

$= \frac{2(4x^2 - 3x - 8)}{(x - 4)(x + 2)}, x \neq -2, 4$

b) $\frac{b}{b^2 + 10b + 24} + \frac{2b}{b^2 + 12b + 32}$

$= \frac{b}{(b + 4)(b + 6)} + \frac{2b}{(b + 4)(b + 8)}$

Common denominator:

$(b + 4)(b + 6)(b + 8)$

$= \frac{b}{(b + 4)(b + 6)} \cdot \frac{(b + 8)}{(b + 8)}$

$+ \frac{2b}{(b + 4)(b + 8)} \cdot \frac{(b + 6)}{(b + 6)}$

$= \frac{b^2 + 8b}{(b + 4)(b + 6)(b + 8)}$

$+ \frac{2b^2 + 12b}{(b + 4)(b + 6)(b + 8)}$

$= \frac{3b^2 + 20b}{(b + 4)(b + 6)(b + 8)}$

$= \frac{b(3b + 20)}{(b + 4)(b + 6)(b + 8)},$

$b \neq -8, -6, -4$

c) $\dfrac{4u^2 - 20u}{u^2 + 2u - 35} - \dfrac{3u - 6}{3u^2 - 10u + 8}$

$= \dfrac{4u\cancel{(u - 5)}}{(u + 7)\cancel{(u - 5)}} - \dfrac{3\cancel{(u - 2)}}{(3u - 4)\cancel{(u - 2)}}$

$= \dfrac{4u}{(u + 7)} - \dfrac{3}{(3u - 4)}$

Common denominator:

$(u + 7)(3u - 4)$

$= \dfrac{4u}{(u + 7)} \cdot \dfrac{(3u - 4)}{(3u - 4)} - \dfrac{3}{(3u - 4)} \cdot \dfrac{(u + 7)}{(u + 7)}$

$= \dfrac{12u^2 - 16u}{(u + 7)(3u - 4)} - \dfrac{3u + 21}{(u + 7)(3u - 4)}$

$= \dfrac{12u^2 - 19u - 21}{(u + 7)(3u - 4)}, u \neq -7, \frac{4}{3}, 2, 5$

12. Simplify.

a) $\dfrac{x + 2}{x^2 + 5x + 6} - \dfrac{2 + x}{4 - x^2} + \dfrac{2 - x}{x^2 + x - 6}$

$= \dfrac{\cancel{x + 2}}{\cancel{(x + 2)}(x + 3)} - \dfrac{\cancel{2 + x}}{(2 - x)\cancel{(2 + x)}} + \dfrac{2 - x}{(x + 3)(x - 2)}$

$= \dfrac{1}{x + 3} - \dfrac{1}{-(x - 2)} + \dfrac{-\cancel{(x - 2)}}{(x + 3)\cancel{(x - 2)}}$

$= \dfrac{1}{x + 3} + \dfrac{1}{x - 2} + \dfrac{-1}{x + 3}$

$= \dfrac{1}{x - 2}, x \neq -3, -2, 2$

b) $\dfrac{x^2 + 3x + 2}{x^2 - 1} + \dfrac{x^2 + x - 2}{x^2 - x} - \dfrac{x^2 - x - 12}{x^2 - 3x - 4}$

$= \dfrac{\cancel{(x + 1)}(x + 2)}{(x - 1)\cancel{(x + 1)}} + \dfrac{(x + 2)\cancel{(x - 1)}}{x\cancel{(x - 1)}} - \dfrac{\cancel{(x - 4)}(x + 3)}{\cancel{(x - 4)}(x + 1)}$

$= \dfrac{x + 2}{x - 1} + \dfrac{x + 2}{x} - \dfrac{x + 3}{x + 1}$

Common denominator:

$x(x - 1)(x + 1)$

$= \dfrac{x + 2}{x - 1} \cdot \dfrac{x(x + 1)}{x(x + 1)} + \dfrac{x + 2}{x} \cdot \dfrac{(x + 1)(x - 1)}{(x + 1)(x - 1)} - \dfrac{x + 3}{x + 1} \cdot \dfrac{x(x - 1)}{x(x - 1)}$

$= \dfrac{(x + 2)(x^2 + x)}{x(x - 1)(x + 1)} + \dfrac{(x + 2)(x^2 - 1)}{x(x - 1)(x + 1)} - \dfrac{(x + 3)(x^2 - x)}{x(x - 1)(x + 1)}$

$= \dfrac{x^3 + x^2 + 2x^2 + 2x + x^3 - x + 2x^2 - 2 - x^3 + x^2 - 3x^2 + 3x}{x(x - 1)(x + 1)}$

$= \dfrac{x^3 + 3x^2 + 4x - 2}{x(x - 1)(x + 1)}, x \neq -1, 0, 1, 4$

13. Simplify.

a) $\frac{2x}{x+3} + \frac{3x}{2x+8} \div \frac{x^2}{3x+12}$

$= \frac{2x}{x+3} + \frac{3\cancel{x}}{2\cancel{(x+4)}} \cdot \frac{3\cancel{(x+4)}}{x^{\cancel{2}}}$

$= \frac{2x}{x+3} + \frac{9}{2x}$

Common denominator:

$2x(x+3)$

$= \frac{2x}{x+3} \cdot \frac{2x}{2x} + \frac{9}{2x} \cdot \frac{(x+3)}{(x+3)}$

$= \frac{4x^2}{2x(x+3)} + \frac{9x+27}{2x(x+3)}$

$= \frac{4x^2+9x+27}{2x(x+3)}, x \neq -4, -3, 0$

b) $\frac{2x^2+16x+14}{3x^2+30x+27} \cdot \frac{x^2+7x-18}{x^2+4x-21} - \frac{x^2}{x^2+x-12}$

$= \frac{2(x^2+8x+7)}{3(x^2+10x+9)} \cdot \frac{(x+9)(x-2)}{(x+7)(x-3)} - \frac{x^2}{(x+4)(x-3)}$

$= \frac{2\cancel{(x+7)}\cancel{(x+1)}}{3\cancel{(x+9)}\cancel{(x+1)}} \cdot \frac{\cancel{(x+9)}(x-2)}{\cancel{(x+7)}(x-3)} - \frac{x^2}{(x+4)(x-3)}$

$= \frac{2(x-2)}{3(x-3)} - \frac{x^2}{(x+4)(x-3)}$

Common denominator:

$3(x-3)(x+4)$

$= \frac{2(x-2)}{3(x-3)} \cdot \frac{(x+4)}{(x+4)} - \frac{x^2}{(x+4)(x-3)} \cdot \frac{3}{3}$

$= \frac{(2x-4)(x+4)}{3(x-3)(x+4)} - \frac{3x^2}{3(x-3)(x+4)}$

$= \frac{2x^2+4x-16-3x^2}{3(x-3)(x+4)}$

$= \frac{-x^2+4x-16}{3(x-3)(x+4)}, x \neq -9, -7, -4, -1, 3$

7.5 Solving Rational Equations

3. Identify the non-permissible values of the variable in each equation.

a) $5 = \frac{3x}{x}$

$x = 0$

b) $\frac{2}{x - 1} = \frac{6}{x}$

$x = 0$ and $x = 1$

c) $\frac{3}{6 - x} = \frac{2}{x + 5}$

$x = -5$ and $x = 6$

d) $\frac{-1}{3x + 6} = \frac{2x}{x - 2}$

$x = -2$ and $x = 2$

4. Solve each equation.

a) $\frac{4}{5} = \frac{8}{d}$

Non-permissible value: $d = 0$
Common denominator: $5d$

$$\cancel{5}d\left(\frac{4}{\cancel{5}}\right) = 5\cancel{d}\left(\frac{8}{\cancel{d}}\right)$$
$$4d = 40$$
$$d = 10$$

b) $\frac{e}{3} = \frac{12}{e}$

Non-permissible value: $e = 0$
Common denominator: $3e$

$$\cancel{3}e\left(\frac{e}{\cancel{3}}\right) = 3\cancel{e}\left(\frac{12}{\cancel{e}}\right)$$
$$e^2 = 36$$
$$e = 6 \text{ or } e = -6$$

Solutions are $e = -6$ and $e = 6$.

5. Solve each equation.

a) $\frac{3}{2z} = \frac{4}{3z} - \frac{1}{2}$

Non-permissible value: $z = 0$
Common denominator: $6z$

$$^3\cancel{6}\cancel{z}\left(\frac{3}{2\cancel{z}}\right) = {}^2\cancel{6}\cancel{z}\left(\frac{4}{3\cancel{z}}\right) - {}^3\cancel{6}z\left(\frac{1}{\cancel{2}}\right)$$
$$9 = 8 - 3z$$
$$3z = -1$$
$$z = -\frac{1}{3}$$

b) $2 - \frac{4}{x} = \frac{6}{x^2}$

Non-permissible value: $x = 0$
Common denominator: x^2

$$x^2(2) - x^{\cancel{2}}\left(\frac{4}{\cancel{x}}\right) = \cancel{x^2}\left(\frac{6}{\cancel{x^2}}\right)$$
$$2x^2 - 4x = 6$$
$$2x^2 - 4x - 6 = 0$$
$$2(x^2 - 2x - 3) = 0$$
$$2(x - 3)(x + 1) = 0$$
$$x = 3 \text{ or } x = -1$$

Solutions are $x = -1$ and $x = 3$.

6. Solve each equation.

a) $\frac{3}{q-2} = \frac{5}{q+4}$

Non-permissible values: $q = 2$ and $q = -4$

Common denominator: $(q - 2)(q + 4)$

$$\cancel{(q-2)}(q+4)\left(\frac{3}{\cancel{q-2}}\right) = (q-2)\cancel{(q+4)}\left(\frac{5}{\cancel{q+4}}\right)$$

$$3q + 12 = 5q - 10$$

$$22 = 2q$$

$$q = 11$$

7. Solve each equation.

a) $\frac{a-1}{a-3} = \frac{a+1}{a-4}$

Non-permissible values: $a = 3$ and $a = 4$

Common denominator: $(a - 3)(a - 4)$

$$\cancel{(a-3)}(a-4)\left(\frac{a-1}{\cancel{a-3}}\right) = (a-3)\cancel{(a-4)}\left(\frac{a+1}{\cancel{a-4}}\right)$$

$$(a-4)(a-1) = (a-3)(a+1)$$

$$a^2 - 5a + 4 = a^2 - 2a - 3$$

$$-3a = -7$$

$$a = \frac{7}{3}$$

c) $\frac{6}{2x^2 + 2x} = \frac{x-2}{x+1}$

$\frac{6}{2x(x+1)} = \frac{x-2}{x+1}$

Non-permissible values: $x = -1$ and $x = 0$

Common denominator: $2x(x + 1)$

$$\cancel{2x(x+1)}\left(\frac{6}{\cancel{2x(x+1)}}\right) = 2x\cancel{(x+1)}\left(\frac{x-2}{\cancel{x+1}}\right)$$

$$6 = 2x^2 - 4x$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

$x = -1$ is a non-permissible value.

So, the only solution is $x = 3$.

9. Solve each equation.

a) $\frac{1}{x} + \frac{1}{x - 3} = \frac{x - 2}{x - 3}$

Non-permissible values: $x = 0$ and $x = 3$

$$\frac{1}{x} = \frac{x - 2}{x - 3} - \frac{1}{x - 3}$$

$$\frac{1}{x} = \frac{x - 3}{x - 3}$$

$$\frac{1}{x} = 1$$

$$x = 1$$

b) $\frac{1}{u - 2} + \frac{u - 1}{u^2 - 4} = \frac{u + 4}{u + 2}$

$$\frac{1}{u - 2} + \frac{u - 1}{(u - 2)(u + 2)} = \frac{u + 4}{u + 2}$$

Non-permissible values: $u = -2$ and $u = 2$

Common denominator: $(u - 2)(u + 2)$

$$\cancel{(u - 2)}(u + 2)\left(\frac{1}{\cancel{u - 2}}\right) + \cancel{(u - 2)(u + 2)}\left(\frac{u - 1}{\cancel{(u - 2)(u + 2)}}\right)$$

$$= (u - 2)\cancel{(u + 2)}\left(\frac{u + 4}{\cancel{u + 2}}\right)$$

$$u + 2 + u - 1 = (u - 2)(u + 4)$$

$$2u + 1 = u^2 + 2u - 8$$

$$u^2 - 9 = 0$$

$$u = 3 \text{ or } u = -3$$

Solutions are $u = -3$ and $u = 3$.

d) $\frac{3z - 1}{2z + 1} + \frac{1}{6} = \frac{2z - 1}{2z + 1} + \frac{z + 1}{z + 3}$

$$\frac{3z - 1}{2z + 1} - \frac{2z - 1}{2z + 1} + \frac{1}{6} = \frac{z + 1}{z + 3}$$

$$\frac{z}{2z + 1} + \frac{1}{6} = \frac{z + 1}{z + 3}$$

Non-permissible values: $z = -\frac{1}{2}$ and $z = -3$

Common denominator: $6(2z + 1)(z + 3)$

$$6\cancel{(2z + 1)}(z + 3)\left(\frac{z}{\cancel{2z + 1}}\right) + \cancel{6}(2z + 1)(z + 3)\left(\frac{1}{\cancel{6}}\right)$$

$$= 6(2z + 1)\cancel{(z + 3)}\left(\frac{z + 1}{\cancel{z + 3}}\right)$$

$$6z(z + 3) + (2z + 1)(z + 3) = (12z + 6)(z + 1)$$

$$6z^2 + 18z + 2z^2 + 7z + 3 = 12z^2 + 18z + 6$$

$$4z^2 - 7z + 3 = 0$$

$$(4z - 3)(z - 1) = 0$$

$$z = \frac{3}{4} \text{ or } z = 1$$

Solutions are $z = \frac{3}{4}$ and $z = 1$.

10. Solve each equation.

a) $\frac{b}{b^2 - 4} = \frac{2}{b^2 - b - 6}$

$$\frac{b}{(b - 2)(b + 2)} = \frac{2}{(b - 3)(b + 2)}$$

Non-permissible values: $b = 2$, $b = -2$, and $b = 3$

Common denominator: $(b - 2)(b + 2)(b - 3)$

$$\cancel{(b - 2)}\cancel{(b + 2)}(b - 3)\left(\frac{b}{\cancel{(b - 2)}\cancel{(b + 2)}}\right)$$

$$= (b - 2)\cancel{(b + 2)}\cancel{(b - 3)}\left(\frac{2}{\cancel{(b - 3)}\cancel{(b + 2)}}\right)$$

$$b^2 - 3b = 2b - 4$$

$$b^2 - 5b + 4 = 0$$

$$(b - 4)(b - 1) = 0$$

$$b = 4 \text{ or } b = 1$$

Solutions are $b = 4$ and $b = 1$.

c) $\frac{n}{n+1} + \frac{3n+5}{n^2+4n+3} = \frac{2}{n+3}$

$\frac{n}{n+1} + \frac{3n+5}{(n+1)(n+3)} = \frac{2}{n+3}$

Non-permissible values: $n = -1$ and $n = -3$

Common denominator: $(n+1)(n+3)$

$$\cancel{(n+1)}(n+3)\left(\frac{n}{\cancel{n+1}}\right) + \cancel{(n+1)(n+3)}\left(\frac{3n+5}{\cancel{(n+1)(n+3)}}\right)$$
$$= (n+1)\cancel{(n+3)}\left(\frac{2}{\cancel{n+3}}\right)$$

$$n^2 + 3n + 3n + 5 = 2n + 2$$
$$n^2 + 4n + 3 = 0$$
$$(n+1)(n+3) = 0$$
$$n = -1 \text{ or } n = -3$$

$n = -1$ and $n = -3$ are non-permissible values.

So, the equation has no solution.

13. Without solving the equation, how do you know that the equation $\frac{12}{4x-4} = \frac{4}{x-1}$ has no solution?

Simplify the expression on the left.

$$\frac{{}^{3}\cancel{12}}{\cancel{4}(x-1)} = \frac{4}{x-1}$$
$$\frac{3}{x-1} = \frac{4}{x-1}$$

Since the expressions have the same denominator but different numerators, I know the equation has no solution.

15. Solve each equation.

a) $\frac{x+1}{x-3} = \frac{2x}{x+2}$

Non-permissible values: $x = 3$ and $x = -2$

Common denominator: $(x-3)(x+2)$

$$\cancel{(x-3)}(x+2)\left(\frac{x+1}{\cancel{x-3}}\right) = (x-3)\cancel{(x+2)}\left(\frac{2x}{\cancel{x+2}}\right)$$
$$x^2 + 3x + 2 = 2x^2 - 6x$$
$$x^2 - 9x - 2 = 0 \qquad \text{Use the quadratic formula.}$$
$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(-2)}}{2(1)}$$
$$x = \frac{9 \pm \sqrt{89}}{2}$$

Solutions are $x = \frac{9+\sqrt{89}}{2}$ and $x = \frac{9-\sqrt{89}}{2}$.

c) $\frac{1}{v} + \frac{1}{v-4} = 2$

Non-permissible values: $v = 0$ and $v = 4$
Common denominator: $v(v-4)$

$$\cancel{v}(v-4)\left(\frac{1}{\cancel{v}}\right) + v\cancel{(v-4)}\left(\frac{1}{\cancel{v-4}}\right) = v(v-4)(2)$$

$$v - 4 + v = 2v^2 - 8v$$

$$2v^2 - 10v + 4 = 0$$

$$v^2 - 5v + 2 = 0 \quad \text{Use the quadratic formula.}$$

$$v = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$v = \frac{5 \pm \sqrt{17}}{2}$$

Solutions are $v = \frac{5 + \sqrt{17}}{2}$ and $v = \frac{5 - \sqrt{17}}{2}$.

7.6 Applications of Rational Equations

5. It takes a painter 3 h to spray paint a fence. When two people paint the fence, one using a sprayer and the other using a brush, they can paint the fence in 2 h. How long would it take one person to paint the fence using only a brush?

Let t hours represent the time it takes one person to paint the fence using only a brush.
After 2 h, a painter has spray painted $\frac{2}{3}$ of the fence and the person using a brush has painted $\frac{2}{t}$ of the fence.

So, an equation is: $\frac{2}{3} + \frac{2}{t} = 1$

Non-permissible value: $t = 0$
Common denominator: $3t$

$$\frac{2}{3} + \frac{2}{t} = 1, t > 0$$

$$\cancel{3}t\left(\frac{2}{\cancel{3}}\right) + 3\cancel{t}\left(\frac{2}{\cancel{t}}\right) = 3t(1)$$

$$2t + 6 = 3t$$

$$t = 6$$

It would take one person 6 h to paint the fence using only a brush.

6. Jenny can clean out the garage in 5 h. When her son helps, they can clean out the garage in 3 h. How long would it take Jenny's son to clean out the garage on his own?

Let t hours represent the time it takes Jenny's son to clean out the garage on his own.
After 3 h, Jenny has cleaned out $\frac{3}{5}$ of the garage and Jenny's son has cleaned out $\frac{3}{t}$ of the garage.
So, an equation is: $\frac{3}{5} + \frac{3}{t} = 1, t > 0$
Non-permissible value: $t = 0$
Common denominator: $5t$

$$\frac{3}{5} + \frac{3}{t} = 1$$
$$\cancel{5}t\left(\frac{3}{\cancel{5}}\right) + 5\cancel{t}\left(\frac{3}{\cancel{t}}\right) = 5t(1)$$
$$3t + 15 = 5t$$
$$2t = 15$$
$$t = 7.5$$

It would take Jenny's son 7.5 h to clean out the garage on his own.

7. How much bleach should be added to 47 L of water to make a solution that is 6% bleach?

Let the volume of bleach added be v litres.
Then, total volume of the solution is $(v + 47)$ litres.

$$\frac{\text{volume of bleach}}{\text{total volume}} = \frac{6}{100}$$
$$\frac{v}{v + 47} = \frac{6}{100}, v > 0$$

$v = -47$ is a non-permissible value.
A common denominator is: $100(v + 47)$

$$100\cancel{(v + 47)}\left(\frac{v}{\cancel{v + 47}}\right) = \cancel{100}(v + 47)\left(\frac{6}{\cancel{100}}\right)$$
$$100v = 6v + 282$$
$$94v = 282$$
$$v = 3$$

To create a solution that is 6% bleach, 3 L of bleach should be added to 47 L of water.

8. A boat travels 4 km upstream in the same time that it takes the boat to travel 10 km downstream. The average speed of the current is 3 km/h. What is the average speed of the boat in still water?

Let the average speed of the boat in still water be s kilometres per hour.
Average speed downstream: $(s + 3)$ km/h
Distance downstream: 10 km
Time downstream: $\frac{10}{s + 3}$ hours
Average speed upstream: $(s - 3)$ km/h
Distance upstream: 4 km
Time upstream: $\frac{4}{s - 3}$ hours
It takes the same time to travel upstream as it does to travel downstream.
So, an equation is: $\frac{10}{s + 3} = \frac{4}{s - 3}$, $s > 3$
$s = 3$ and $s = -3$ are non-permissible values.
A common denominator is: $(s + 3)(s - 3)$

$$\cancel{(s + 3)}(s - 3)\left(\frac{10}{\cancel{s + 3}}\right) = (s + 3)\cancel{(s - 3)}\left(\frac{4}{\cancel{s - 3}}\right)$$

$$10s - 30 = 4s + 12$$
$$6s = 42$$
$$s = 7$$

The average speed of the boat in still water is 7 km/h.

9. A natural number is 4 more than another natural number. When the reciprocal of the greater number is subtracted from the reciprocal of the lesser number, the difference is $\frac{1}{15}$. What are the two numbers?

Let one natural number be x. Then the other natural number is $x + 4$.
The reciprocal of the lesser number is: $\frac{1}{x}$

The reciprocal of the greater number is: $\frac{1}{x + 4}$
Their difference is: $\frac{1}{15}$

So, an equation is: $\frac{1}{x} - \frac{1}{x + 4} = \frac{1}{15}$, $x \in \mathbb{N}$

$x = -4$ and $x = 0$ are non-permissible values.
A common denominator is: $15(x)(x + 4)$

$$15(\cancel{x})(x + 4)\left(\frac{1}{\cancel{x}}\right) - 15(x)\cancel{(x + 4)}\left(\frac{1}{\cancel{x + 4}}\right) = \cancel{15}(x)(x + 4)\left(\frac{1}{\cancel{15}}\right)$$

$$15x + 60 - 15x = x^2 + 4x$$
$$x^2 + 4x - 60 = 0$$
$$(x + 10)(x - 6) = 0$$

$x = -10$ or $x = 6$
Since $x \in \mathbb{N}$, $x = -10$ is not a solution.
So, the natural numbers are 6 and $6 + 4$, or 10.

10. It takes Marcy's apprentice 9 h longer to build a deck than it takes Marcy, an experienced carpenter. When they work together, they can build the deck in 20 h. How long would it take each person to build the deck working alone?

Let t hours represent the time it takes Marcy to build a deck.
Then, the time it takes Marcy's apprentice is $(t + 9)$ hours.
After 20 h, Marcy has built $\frac{20}{t}$ of the deck and Marcy's apprentice has built $\frac{20}{t + 9}$ of the deck.
So, an equation is: $\frac{20}{t} + \frac{20}{t + 9} = 1, t > 0$
Non-permissible values: $t = 0$ and $t = -9$
Common denominator: $t(t + 9)$

$$\frac{20}{t} + \frac{20}{t + 9} = 1$$

$$\cancel{t}(t + 9)\left(\frac{20}{\cancel{t}}\right) + t\cancel{(t + 9)}\left(\frac{20}{\cancel{t + 9}}\right) = t(t + 9)(1)$$

$$20t + 180 + 20t = t^2 + 9t$$

$$t^2 - 31t - 180 = 0$$

$$(t - 36)(t + 5) = 0$$

$t = 36$ or $t = -5$
Since time cannot be negative, $t = 36$
It would take Marcy 36 h to build the deck and it would take Marcy's apprentice 36 h + 9 h, or 45 h to build the deck.

11. The average speed of an airplane is 10 times that of a car. It takes the airplane 18 h less than the car to travel 1000 km. Determine the average speeds of the airplane and the car.

Let the average speed of the car be s kilometres per hour.
Then the average speed of the airplane is $10s$ kilometres per hour.
Distance: 1000 km
Time for car: $\frac{1000}{s}$ hours
Time for airplane: $\frac{1000}{10s}$ hours
It takes the airplane 18 h less than the car to travel this distance.
So, an equation is: $\frac{1000}{s} - \frac{1000}{10s} = 18,\ s > 0$
Non-permissible value: $s = 0$
Common denominator: $10s$

$$\frac{1000}{s} - \frac{1000}{10s} = 18$$

$$10\cancel{s}\left(\frac{1000}{\cancel{s}}\right) - \cancel{10s}\left(\frac{1000}{\cancel{10s}}\right) = 10s(18)$$

$$10\,000 - 1000 = 180s$$

$$9000 = 180s$$

$$s = 50$$

The average speed of the car is 50 km/h and the average speed of the airplane is 10(50 km/h), or 500 km/h.

12. Ann cycles 6 km to return a friend's bicycle. She then walks home. Her total time for the trip is 90 min. Ann cycles four times as fast as she walks. Determine Ann's average speeds for walking and for cycling.

Let Ann's average walking speed be s kilometres per hour.
Then her average cycling speed is $4s$ kilometres per hour.
Distance: 6 km
Time cycling: $\frac{6}{s}$ hours
Time walking: $\frac{6}{4s}$ hours
Total time taken is 90 min, or 1.5 h.
So, an equation is: $\frac{6}{s} + \frac{6}{4s} = 1.5,\ s > 0$
Non-permissible value: $s = 0$
Common denominator: $4s$

$$\frac{6}{s} + \frac{6}{4s} = 1.5$$

$$4\cancel{s}\left(\frac{6}{\cancel{s}}\right) + \cancel{4s}\left(\frac{6}{\cancel{4s}}\right) = 4s(1.5)$$

$$24 + 6 = 6s$$

$$30 = 6s$$

$$s = 5$$

Ann's average walking speed is 5 km/h and her average cycling speed is 4(5 km/h), or 20 km/h.

Review

1. For each rational expression below:

- Identify the non-permissible values.
- Write the expressions in simplest form.
- Write an equivalent expression.

a) $\frac{2x + 16}{x^2 - 64} = \frac{2(x + 8)}{(x + 8)(x - 8)}$

Non-permissible values: $x = -8$ and $x = 8$

$$\frac{2\cancel{(x + 8)}}{\cancel{(x + 8)}(x - 8)} = \frac{2}{x - 8},\ x \neq -8, 8$$

$$\frac{2(x + 8)}{(x + 8)(x - 8)} = \frac{2(x + 8)}{(x + 8)(x - 8)} \cdot \frac{x + 8}{x + 8}$$

$$= \frac{2(x + 8)^2}{(x + 8)^2(x - 8)},\ x \neq -8, 8$$

b) $\dfrac{x^2 + x - 2}{-x^2 - 2x + 3} = \dfrac{(x + 2)(x - 1)}{-(x^2 + 2x - 3)}$

$= \dfrac{(x + 2)(x - 1)}{-(x + 3)(x - 1)}$

Non-permissible values: $x = -3$ and $x = 1$

$\dfrac{(x + 2)\cancel{(x - 1)}}{-(x + 3)\cancel{(x - 1)}} = \dfrac{x + 2}{-x - 3}, x \neq -3, 1$

$\dfrac{(x + 2)(x - 1)}{-(x + 3)(x - 1)} = \dfrac{(x + 2)(x - 1)}{-(x + 3)(x - 1)} \cdot \dfrac{x + 2}{x + 2}$

$= \dfrac{(x + 2)^2(x - 1)}{-(x + 3)(x - 1)(x + 2)}, x \neq -3, -2, 1$

2. Simplify each expression.

a) $\dfrac{25ab^2}{4b} \cdot \dfrac{10b^3}{5a}$

$= \dfrac{{}^{5}\cancel{25}\,\cancel{a}\,b^{\cancel{2}}}{{}_{2}\cancel{4}\,\cancel{b}} \cdot \dfrac{\cancel{10}^{\cancel{5}}b^3}{\cancel{5}\,\cancel{a}}$

$= \dfrac{25b^4}{2}, a \neq 0, b \neq 0$

b) $\dfrac{6y^2}{y^2 - 9} \cdot \dfrac{y + 3}{15y}$

$= \dfrac{6y^2}{(y - 3)(y + 3)} \cdot \dfrac{y + 3}{15y}$

$= \dfrac{{}^{2}\cancel{6}\,y^{\cancel{2}}}{(y - 3)\cancel{(y + 3)}} \cdot \dfrac{\cancel{y + 3}}{{}_{5}\cancel{15}\,\cancel{y}}$

$= \dfrac{2y}{5(y - 3)}, y \neq -3, 0, 3$

c) $\dfrac{n - 2}{n} \div \dfrac{3n - 6}{n^2 + n}$

$= \dfrac{n - 2}{n} \div \dfrac{3(n - 2)}{n(n + 1)}$

$= \dfrac{\cancel{n - 2}}{\cancel{n}} \cdot \dfrac{\cancel{n}(n + 1)}{3\cancel{(n - 2)}}$

$= \dfrac{n + 1}{3}, n \neq -1, 0, 2$

d) $\dfrac{b^2 + 4b + 3}{3 - b} \div \dfrac{b^2 + 2b + 1}{b^2 - 9}$

$= \dfrac{(b + 1)(b + 3)}{3 - b} \div \dfrac{(b + 1)(b + 1)}{(b - 3)(b + 3)}$

$= \dfrac{(b + 1)(b + 3)}{3 - b} \cdot \dfrac{(b - 3)(b + 3)}{(b + 1)(b + 1)}$

$= \dfrac{\cancel{(b + 1)}(b + 3)}{-\cancel{(b - 3)}} \cdot \dfrac{\cancel{(b - 3)}(b + 3)}{\cancel{(b + 1)}(b + 1)}$

$= -\dfrac{(b + 3)^2}{b + 1}, b \neq -3, -1, 3$

3. Add or subtract.

a) $\frac{6}{x^3} + \frac{5}{2x^4}$

Common denominator: $2x^4$

$= \frac{6}{x^3} \cdot \frac{2x}{2x} + \frac{5}{2x^4}$

$= \frac{12x}{2x^4} + \frac{5}{2x^4}$

$= \frac{12x + 5}{2x^4}, x \neq 0$

b) $\frac{-8}{3ef} - \frac{2}{4f} + \frac{5}{2e^2}$

Common denominator: $12e^2f$

$= \frac{-8}{3ef} \cdot \frac{4e}{4e} - \frac{2}{4f} \cdot \frac{3e^2}{3e^2} + \frac{5}{2e^2} \cdot \frac{6f}{6f}$

$= \frac{-32e}{12e^2f} - \frac{6e^2}{12e^2f} + \frac{30f}{12e^2f}$

$= \frac{-32e - 6e^2 + 30f}{12e^2f}$

$= \frac{2(-16e - 3e^2 + 15f)}{12e^2f}$

$= \frac{-16e - 3e^2 + 15f}{6e^2f}, e \neq 0, f \neq 0$

4. Add or subtract.

a) $\frac{6}{v - 2} + \frac{7}{2v + 7}$

Common denominator: $(v - 2)(2v + 7)$

$= \frac{6}{(v - 2)} \cdot \frac{(2v + 7)}{(2v + 7)} + \frac{7}{(2v + 7)} \cdot \frac{(v - 2)}{(v - 2)}$

$= \frac{12v + 42}{(v - 2)(2v + 7)} + \frac{7v - 14}{(v - 2)(2v + 7)}$

$= \frac{19v + 28}{(v - 2)(2v + 7)}, v \neq -\frac{7}{2}, 2$

b) $\frac{2z}{3z^2 + 19z - 14} + \frac{3}{9z^2 - 12z + 4}$

$= \frac{2z}{(3z - 2)(z + 7)} + \frac{3}{(3z - 2)(3z - 2)}$

Common denominator: $(3z - 2)^2(z + 7)$

$= \frac{2z}{(3z - 2)(z + 7)} \cdot \frac{(3z - 2)}{(3z - 2)} + \frac{3}{(3z - 2)(3z - 2)} \cdot \frac{(z + 7)}{(z + 7)}$

$= \frac{6z^2 - 4z}{(3z - 2)^2(z + 7)} + \frac{3z + 21}{(3z - 2)^2(z + 7)}$

$= \frac{6z^2 - z + 21}{(3z - 2)^2(z + 7)}, z \neq -7, \frac{2}{3}$

5. Solve each equation.

a) $\frac{3}{5f} = 1 - \frac{7}{2f}$

Non-permissible value: $f = 0$

Common denominator: $10f$

$$\frac{3}{5f} = 1 - \frac{7}{2f}$$

$$\frac{3}{5f} = \frac{2f - 7}{2f}$$

$$^{2}\cancel{10f}\left(\frac{3}{\cancel{5f}}\right) = {}^{5}\cancel{10f}\left(\frac{2f - 7}{\cancel{2f}}\right)$$

$$6 = 10f - 35$$

$$41 = 10f$$

$$f = \frac{41}{10}$$

b) $\frac{4}{5x - 2} = \frac{3}{4x - 1}$

Non-permissible values:

$x = \frac{2}{5}$ and $x = \frac{1}{4}$

Common denominator:

$(5x - 2)(4x - 1)$

$$\cancel{(5x - 2)}(4x - 1)\left(\frac{4}{\cancel{5x - 2}}\right)$$

$$= (5x - 2)\cancel{(4x - 1)}\left(\frac{3}{\cancel{4x - 1}}\right)$$

$$16x - 4 = 15x - 6$$

$$x = -2$$

c) $\frac{2p}{p - 1} + \frac{p - 5}{p^2 - 1} = 1$

$$\frac{2p}{p - 1} + \frac{p - 5}{(p - 1)(p + 1)} = 1$$

Non-permissible values: $p = 1$ and $p = -1$

Common denominator: $(p - 1)(p + 1)$

$$\cancel{(p - 1)}(p + 1)\left(\frac{2p}{\cancel{p - 1}}\right) + \cancel{(p - 1)}\cancel{(p + 1)}\left(\frac{p - 5}{\cancel{(p - 1)}\cancel{(p + 1)}}\right) = (p - 1)(p + 1)(1)$$

$$2p^2 + 2p + p - 5 = p^2 - 1$$

$$p^2 + 3p - 4 = 0$$

$$(p + 4)(p - 1) = 0$$

$$p = -4 \text{ or } p = 1$$

$p = 1$ is a non-permissible value.

So, the only solution is $p = -4$.

6. Two fishing boats have the same average speed in still water. They leave a dock at the same time. One boat heads upstream and the other heads downstream. At a certain point, boat A is 56 km downstream and boat B is 24 km upstream. The average speed of the current is 8 km/h. What is the average speed of the boats in still water?

Let the average speed of the boats in still water be s km/h.
Average speed of current: 8 km/h
Boat A: downstream
Average speed downstream: $(s + 8)$ km/h
Distance travelled: 56 km
Time for boat A: $\frac{56}{s + 8}$ hours
Boat B: upstream
Average speed upstream: $(s - 8)$ km/h
Distance travelled: 24 km
Time for boat B: $\frac{24}{s - 8}$ hours
Boats take same time to travel these distances.
So, an equation is: $\frac{56}{s + 8} = \frac{24}{s - 8}$, $s > 8$
Non-permissible values: $s = -8$ and $s = 8$
Common denominator: $(s + 8)(s - 8)$

$$\frac{56}{s + 8} = \frac{24}{s - 8}$$

$$\cancel{(s + 8)}(s - 8)\left(\frac{56}{\cancel{s + 8}}\right) = (s + 8)\cancel{(s - 8)}\left(\frac{24}{\cancel{s - 8}}\right)$$

$$56s - 448 = 24s + 192$$

$$32s = 640$$

$$s = 20$$

The average speed of the boats in still water is 20 km/h.