

Chapter 2 ReviewName Key

1. Evaluate the following:

$$\begin{aligned} \text{a) } & 3|-8+4| - 5|3 - (-2)| \\ & = 3|-4| - 5|5| \\ & = 3(4) - 5(5) \\ & = 12 - 25 \\ & = -13 \end{aligned}$$

$$\begin{aligned} \text{b) } & 4|5-2| + 3|5-8| \\ & = 4|3| + 3|-3| \\ & = 4(3) + 3(3) \\ & = 12 + 9 \\ & = 21 \end{aligned}$$

2. For which values of the variable,
- x
- , are the following radicals defined:

a) $\sqrt{8x}$

$x \geq 0$

Can only take square root of positive.

b) $\sqrt{-8x^5}$

$x \leq 0$

Neg $8x$ \times Neg = Pos.
Can only sq. rt a positive

c) $\sqrt[3]{16x}$

$x \in \mathbb{R}$

Can take cube root of positive or negative

3. Express the following mixed radicals as entire radicals:

$$\begin{aligned} \text{a) } & 3\sqrt{5} \\ & = \sqrt{3^2} \sqrt{5} \\ & = \sqrt{9} \sqrt{5} \\ & = \sqrt{45} \end{aligned}$$

$$\begin{aligned} \text{b) } & (2)\sqrt[4]{3} \\ & = \sqrt[4]{2^4} \sqrt[4]{3} \\ & = \sqrt[4]{16 \cdot 3} \\ & = \sqrt[4]{48} \end{aligned}$$

$$\begin{aligned} \text{c) } & (3x)\sqrt[3]{2} \\ & = \sqrt[3]{(3x)^3} \sqrt[3]{2} \\ & = \sqrt[3]{27x^3} \sqrt[3]{2} \\ & = \sqrt[3]{54x^3} \end{aligned}$$

$$\begin{aligned} \text{d) } & 4m^2n\sqrt{3} \\ & = \sqrt{(4m^2n)^2} \sqrt{3} \\ & = \sqrt{16m^4n^2} \sqrt{3} \\ & = \sqrt{48m^4n^2} \end{aligned}$$

$$\begin{aligned} \text{e) } & (-2)\sqrt[4]{6xy^3} \\ & = \sqrt[4]{(-2)^4} \sqrt[4]{6xy^3} \\ & = \sqrt[4]{16} \sqrt[4]{6xy^3} \\ & = \sqrt[4]{96xy^3} \end{aligned}$$

$$\begin{aligned} \text{f) } & (2m^4)\sqrt[5]{3m^2} \\ & = \sqrt[5]{(2m^4)^5} \sqrt[5]{3m^2} \\ & = \sqrt[5]{32m^9} \sqrt[5]{3m^2} \\ & = \sqrt[5]{64m^{11}} \end{aligned}$$

4. Express the following entire radicals as mixed radicals

$$\begin{aligned} \text{a) } & \sqrt{162} \\ & = \sqrt{81 \cdot 2} \\ & = \sqrt{81} \sqrt{2} \\ & = 9\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{b) } & \sqrt[3]{72} \\ & = \sqrt[3]{8 \cdot 9} \\ & = 2\sqrt[3]{9} \end{aligned}$$

$$\begin{aligned} \text{c) } & \sqrt{\frac{32}{50}} \\ & = \sqrt{\frac{16 \cdot 2}{25 \cdot 2}} \\ & = \frac{4\sqrt{2}}{5\sqrt{2}} = \frac{4}{5} \end{aligned}$$

$$\begin{aligned}
 d) & \sqrt[3]{16x^3y} \\
 & = \sqrt[3]{8 \cdot 2} \sqrt[3]{x^3} \sqrt[3]{y} \\
 & = 2\sqrt[3]{2} x \sqrt[3]{y} \\
 & = 2x \sqrt[3]{2y}
 \end{aligned}$$

$$\begin{aligned}
 e) & \sqrt{2m^5n^4} \\
 & = \sqrt{2} \sqrt{mmmm} \sqrt{nnnn} \\
 & = \sqrt{2} \sqrt{m} \sqrt{m} \sqrt{n} \sqrt{n} \sqrt{n} \sqrt{n} \\
 & = \sqrt{2} (m)(m) \sqrt{m} (n)(n) \\
 & = m^2 n^2 \sqrt{2m}
 \end{aligned}$$

$$\begin{aligned}
 f) & \sqrt[3]{\frac{375n^6}{24m^3}} \\
 & = \frac{\sqrt[3]{125 \cdot 3}}{\sqrt[3]{8 \cdot 3}} \frac{\sqrt[3]{n^3 n^3 n^3}}{\sqrt[3]{m^3 m^3 m^3}} \\
 & = \frac{5\cancel{\sqrt[3]{3}} n n}{2\cancel{\sqrt[3]{3}} m} = \frac{5n^2}{2m}
 \end{aligned}$$

5. Simplify the following. Answer in simplest form.

$$\begin{aligned}
 a) & \sqrt{7} - \sqrt{28} + 3\sqrt{63} \\
 & = \sqrt{7} - \sqrt{4 \cdot 7} + 3\sqrt{9 \cdot 7} \\
 & = \sqrt{7} - 2\sqrt{7} + 3(3)\sqrt{7} \\
 & = -1\sqrt{7} + 9\sqrt{7} \\
 & = 8\sqrt{7} \\
 c) & \sqrt[4]{48} - \frac{2}{3}\sqrt[4]{243} \\
 & = \sqrt[4]{16 \cdot 3} - \frac{2}{3}\sqrt[4]{81 \cdot 3} \\
 & = 2\sqrt[4]{3} - \frac{2}{3}(3)\sqrt[4]{3} \\
 & = 2\sqrt[4]{3} - 2\sqrt[4]{3} \\
 & = 0
 \end{aligned}$$

$$\begin{aligned}
 e) & 3\sqrt{3x^3} - 3\sqrt{12x^3} \\
 & = 3\sqrt{3}\sqrt{(xx)x} - 3\sqrt{4 \cdot 3}\sqrt{(xx)x} \\
 & = 3\sqrt{3}x\sqrt{x} - 3(2)\sqrt{3}x\sqrt{x} \\
 & = 3x\sqrt{3x} - 6x\sqrt{3x} \\
 & = -3x\sqrt{3x}
 \end{aligned}$$

$$\begin{aligned}
 g) & \frac{\sqrt{64n^3}}{2} - \sqrt{9n^3} + \frac{1}{5}\sqrt{25n^5} \\
 & = \frac{8\sqrt{nnn}}{2} - 3\sqrt{nnn} + \frac{1}{5}(5)\sqrt{nnnnn} \\
 & = 4n\sqrt{n} - 3n\sqrt{n} + 1nn\sqrt{n} \\
 & = 4n\sqrt{n} - 3n\sqrt{n} + n^2\sqrt{n} \\
 & = n\sqrt{n} + n^2\sqrt{n}
 \end{aligned}$$

$$\begin{aligned}
 b) & 3\sqrt{175} - 6\sqrt{32} + \sqrt{98} \\
 & = 3\sqrt{25 \cdot 7} - 6\sqrt{16 \cdot 2} + \sqrt{49 \cdot 2} \\
 & = 3(5)\sqrt{7} - 6(4)\sqrt{2} + 7\sqrt{2} \\
 & = 15\sqrt{7} - 24\sqrt{2} + 7\sqrt{2} \\
 & = 15\sqrt{7} - 17\sqrt{2} \\
 d) & (2)\sqrt[3]{16} + \sqrt[3]{375} - \sqrt[3]{54} + (3)\sqrt[3]{24} \\
 & = 2\sqrt[3]{8 \cdot 2} + \sqrt[3]{125 \cdot 3} - \sqrt[3]{27 \cdot 2} + 3\sqrt[3]{8 \cdot 3} \\
 & = 2(2)\sqrt[3]{2} + 5\sqrt[3]{3} - 3\sqrt[3]{2} + 3(2)\sqrt[3]{3} \\
 & = \underline{4\sqrt[3]{2}} + 5\sqrt[3]{3} - \underline{3\sqrt[3]{2}} + 6\sqrt[3]{3} \\
 & = \sqrt[3]{2} + 11\sqrt[3]{3}
 \end{aligned}$$

$$\begin{aligned}
 f) & \sqrt{32a^2b^3} - ab\sqrt{98b} \\
 & = \sqrt{16 \cdot 2} \sqrt{a^2} \sqrt{bbb} - ab\sqrt{49 \cdot 2} \sqrt{b} \\
 & = 4\sqrt{2} a b \sqrt{b} - ab(7)\sqrt{2} \sqrt{b} \\
 & = 4ab\sqrt{2b} - 7ab\sqrt{2b} \\
 & = -3ab\sqrt{2b}
 \end{aligned}$$

6. Simplify the following. Answer in simplest form.

a) $(6\sqrt{3})(5\sqrt{2})$

$$= 30\sqrt{6}$$

b) $(4\sqrt{18a^2})(\sqrt{3a^2})$

$$= 4\sqrt{9 \cdot 2} \sqrt{a^2} \sqrt{3} \sqrt{a^2}$$

$$= 4(3)\sqrt{2} a \sqrt{3} a = 12a^2\sqrt{6}$$

c) $4\sqrt{5}(2\sqrt{80} - 3\sqrt{45})$

$$= 8\sqrt{400} - 12\sqrt{225}$$

$$= 8(20) - 12(15)$$

$$= 160 - 180$$

$$= -20$$

e) $(5 - 4\sqrt{3})(-2 + \sqrt{3})$

$$= -10 + 5\sqrt{3} + 8\sqrt{3} - 4(3)$$

$$= -10 + 5\sqrt{3} + 8\sqrt{3} - 12$$

$$= -22 + 13\sqrt{3}$$

d) $2\sqrt{5}(\sqrt{6} + \sqrt{2})$

$$= 2\sqrt{30} + 2\sqrt{10}$$

f) $((8)(\sqrt[3]{4c^2}) - c)(\sqrt[3]{2c} + 5c)$

$$= 8\sqrt[3]{8c^3} + 40c\sqrt[3]{4c^2} - c\sqrt[3]{2c} - 5c^2$$

$$= 8(2)c + 40c\sqrt[3]{4c^2} - c\sqrt[3]{2c} - 5c^2$$

$$= 16c + 40c\sqrt[3]{4c^2} - c\sqrt[3]{2c} - 5c^2$$

g) $(-2 - 3\sqrt{6})^2$

$$= (-2 - 3\sqrt{6})(-2 - 3\sqrt{6})$$

$$= 4 + 6\sqrt{6} + 6\sqrt{6} + 9(6)$$

$$= 4 + 12\sqrt{6} + 54$$

$$= 58 + 12\sqrt{6}$$

h) $(\sqrt{2} - 3\sqrt{5m})^2$

$$= (\sqrt{2} - 3\sqrt{5m})(\sqrt{2} - 3\sqrt{5m})$$

$$= 2 - 3\sqrt{10m} - 3\sqrt{10m} - 9(5m)$$

$$= 2 - 6\sqrt{10m} - 45m$$

7. Simplify the following. Answer in simplest form. (Note that this means you must rationalize the denominator if need be...)

a) $\frac{-5\sqrt{80}}{\sqrt{5}}$

$$= -5\sqrt{16}$$

$$= -5(4)$$

$$= -20$$

b) $\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$$= \frac{4\sqrt{5}}{5}$$

c) $\frac{72\sqrt{45}}{-24\sqrt{20}}$

$$= \frac{-3\sqrt{9 \cdot 5}}{1\sqrt{4 \cdot 5}}$$

$$= \frac{-3(3)\sqrt{5}}{1(2)\sqrt{5}}$$

$$= -\frac{9}{2}$$

d) $\frac{3\sqrt{6}}{2\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$

$$\begin{aligned} &= \frac{3\sqrt{90}}{2(15)} \\ &= \frac{3\sqrt{9 \cdot 10}}{30} \\ &= \frac{1(3)\sqrt{10}}{10} = \frac{3\sqrt{10}}{10} \\ \text{g) } &\frac{2\sqrt{5}}{(7+\sqrt{3})} \frac{(7-\sqrt{3})}{(7-\sqrt{3})} \\ &= \frac{14\sqrt{5}-2\sqrt{15}}{49-7\sqrt{3}+7\sqrt{3}-3} \\ &= \frac{14\sqrt{5}-2\sqrt{15}}{46} \\ &= \frac{7\sqrt{5}-\sqrt{15}}{23} \end{aligned}$$

e) $\frac{-3\sqrt{3a}}{4\sqrt{8a}}$

$$\begin{aligned} &= \frac{-3\sqrt{3a}}{4\sqrt{4 \cdot 2a}} \\ &= \frac{-3\sqrt{3a}}{4(2)\sqrt{2a}} \cdot \frac{\sqrt{2a}}{\sqrt{2a}} \\ &= \frac{-3\sqrt{6a^2}}{8(2a)} \\ \text{f) } &\frac{\sqrt{15xy}}{\sqrt{10xy}} \cdot \frac{\sqrt{10xy}}{\sqrt{10xy}} \\ &= \frac{\sqrt{150xy}}{10xy} \\ &= \frac{\sqrt{25 \cdot 6}}{10} \\ &= \frac{5\sqrt{6}}{10} = \frac{\sqrt{6}}{2} \end{aligned}$$

h) $\frac{(2+\sqrt{3})}{(5\sqrt{2}-\sqrt{5})} \frac{(5\sqrt{2}+\sqrt{5})}{(5\sqrt{2}+\sqrt{5})}$

$$\begin{aligned} &= \frac{10\sqrt{2}+2\sqrt{5}+5\sqrt{6}+\sqrt{15}}{25(2)+5\sqrt{10}-5\sqrt{10}-5} \\ &= \frac{10\sqrt{2}+2\sqrt{5}+5\sqrt{6}+\sqrt{15}}{50-5} = \frac{10\sqrt{2}+2\sqrt{5}+5\sqrt{6}+\sqrt{15}}{45} \end{aligned}$$

8. Solve the following.

a) $\sqrt{m-1} + 7 = 13$

$$\begin{array}{rcl} -7 & -7 \\ (\sqrt{m-1})^2 & = (6)^2 \\ m-1 & = 36 \end{array}$$

$$\begin{array}{rcl} +1 & +1 \\ m & = 37 \end{array}$$

Restrictions:
 $m-1 \geq 0$
 $\boxed{m \geq 1}$

b) $8 + \sqrt{5a-5} = -3$

$$\begin{array}{rcl} -8 & -8 \\ (\sqrt{5a-5})^2 & = (-11)^2 \\ 5a-5 & = 121 \end{array}$$

$$\begin{array}{rcl} +5 & +5 \\ \frac{5a}{5} & = \frac{126}{5} \\ a & = 25.2 \end{array}$$

Restrictions:
 $5a-5 \geq 0$
 $5a \geq 5$
 $\boxed{a \geq 1}$

c) $(\sqrt{3n})^2 = (\sqrt{4n-1})^2$

$$\begin{array}{rcl} 3n & = 4n-1 \\ -4n & -4n \\ -1n & = -1 \\ -1 & -1 \\ n & = 1 \end{array}$$

Restrictions
 $3n \geq 0$
 $n \geq 0$
 $4n-1 \geq 0$
 $4n \geq 1$
 $n \geq \frac{1}{4}$
 $\boxed{n \geq \frac{1}{4}}$

d) $\left(\frac{x}{10}\right)^2 = (\sqrt{3x-58})^2$

$$\begin{array}{rcl} 10 \cdot \frac{x}{10} & = (3x-58) \cdot 10 \\ x & = 30x-580 \end{array}$$

$$\begin{array}{rcl} -30x & -30x \\ -29x & = -580 \\ -29 & -29 \end{array}$$

Restrictions

$$\begin{array}{l} \frac{x}{10} > 0 \quad 3x-58 \geq 0 \\ x \geq 0 \quad \frac{3x}{3} \geq \frac{58}{3} \\ x \geq 19.3 \end{array}$$

$$x = 20$$